

SIX LESSONS
To the PROFESSORS of the
MATHEMATIQUES.
ONE OF
GEOMETRY,
THE OTHER OF
ASTRONOMY:

In the Chaires set up by the Noble and Learned Sir HENRY SAVILE, in the
University of Oxford.



LONDON,
Printed by J. M. for Andrew Crook, at
the Green-Dragon, in Pauls
Church-yard.

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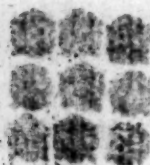
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To the Right Honourable,
Henry Lord Pierrepont,
Viscount Newarke, Earle of Kingstone,
and Marquis of Dorchester,

My most Noble Lord,

 Of knowing on my own part any cause of the favour your Lordship has been pleased to express towards me, unless it be the Principles, Method, and Manners you have observed and approved in my Writings; and seeing these have all been very much reprehended by men to whom the name of Publicque Professors hath procured reputation in the Univerſity of Oxford, I thought it would be a forfeiture of your Lordships good opinion, not to justify my self in publique also against them. Which, whether I have sufficiently performed or not in the six following Lessons addressed to the same Professors, I humbly pray your Lordship to consider. The volume it self is too small to be offered to you as a Present; but to be brought before you as a Controversie it is perhaps the better for being short. Of Arts, some are demonstrable, others indemonstrable; and demonstrable are those the construction of the Subject whereof is in the power of the Artist himself; who in his demonstration does no more but deduce the Consequences of his own operation. The reason whereof is this, that the Science of every Subject is derived from a præcognition of the Causes, Generation, and Construction of the same; and consequently where the Causes are known, there is place for Demonstration; but not where the Causes are to seek for. Geometry therefore is demonstrable; for the Lines and Figures from which we reason are drawn and described by our selves; and Cl-

The Epistle Dedicatory.

vill Philosophy, ~~and demonstration~~ ~~the~~ ~~Common-~~
wealth our selves. But because of Naturall Bodies we know
not the Construction, but see it from the Effects, there lyes no
demonstration of what the Causes be we seek for, but onely of
what they may be.

And where there is place for Demonstration, if the first Prin-
ciples, that is to say, the Definitions contain nor the Generation
of the Subject, there can be nothing demonstrated as it ought
to be. And this in the three first Definitions of *Euclide* suffici-
ently appeareth. For seeing he maketh not, nor could make
any use of them in his Demonstrations, they ought not to be
numbered among the Principles of Geometry. And *Sextus Em-
piricus* maketh use of them (misunderstood, yet so understood
as the said Professors understod them) to the overthrow of that
so much renowned Evidence of Geometry. In that part therefore
of my Book where I treat of Geometry, I thought it necessary
in my Definitions to express those Motions by which Lines, Sur-
pericies, Solids and Figures were drawn and described, little
expecting that any Professor of Geometry should finde fault
therewith; but on the contrary supposing I might thereby not
only avoid the Cavils of the Scepticks, but also demonstrate di-
vers Propositions which on other Principles are indemonstrable.
And truly, if you shall finde those my Principles of Motion made
good, you shall finde also that I have added something to that
which was formerly extant in Geometry.

For first from the seventh Chapter of my Book *de Corpore* to
the thirteenth, I have rectified and explained the Principles of
the Science, *id est*, I have done that business for which *Doctor
Wallis* receives the wages. In the seventh, I have exhibited and
demonstrated the proportion of the Parabola and Parabola-
sters to the Parallelograms of the same height and base,
which (though some of the propositions were extant without
their demonstration) were never before demonstrated, nor are by
any other then this method demonstrable.

In the eighteenth, (as it is now in English) I have demonstra-
ted the (for any thing I yet perceive) Equation between the crook-
ed line of a Parabola or any Parabolaster and a straight line.

The Epistle Dedicatory.

In the twenty-third, I have exhibited the Center of Gravity of any Sector of a Sphere.

Lastly, the twenty-fourth, (which is of the nature of Refraction and Reflexion) is almost all new.

But your Lordship will ask me what I have done in the twentieth, about the Quadrature of the Circle. Truly, my Lord, not much more then before. I have let stand there that which I did before condemn, not that I think it exact, but partly because the Division of Angles may be more exactly performed by it then by any organicall way whatsoever; and I have attempted the same by another Method, which seemeth to me very naturall, but of calculation difficult and slippery. I call them only Aggressions, retaining nevertheless the formall manner of Assertion used in Demonstration. For I dare not use such a doubtfull word as *Videtur*, because the Professors are presently ready to oppose me with a *Videtur quod non*. Nor am I willing to leave those Aggressions out, but rather to try if it may be made pass for lawfull (in spite of them that seek honour not from their own performances but from other mens failings) amongst many difficult undertakings carryed through at once) to leave one and the greatest for a time behind; and partly because the method is such as may hereafter give further light to the finding out of the exact truth.

But the Principles of the Professors that reprehend these of mine, are some of them so void of sense, that a man at the first hearing, whether Geometrician or not Geometrician must abhor them. As for example;

1. That two equall Proportions are not double to one of the same Proportions.
2. That a Proportion is double, triple, &c. of a Number, but not of a Proportion.
3. That the same Body, without adding to it, or taking from it, is sometimes Greater, and sometimes less.
4. That a Quantity may grow less and less Eternally, so as at last to be equall to another Quantity; or (which is all one) that there is a Last in Eternity.
5. That the nature of an Angle consisteth in that which lyes between

between the lines that comprehend the Angle in the very point of their concourie; that is to say, An Angle is the Superficies which lyes between the two Points which touch, or (as they understand a Point) the Superficies that lyes between the two Nothings which touch.

6. That the Quorient is the Proportion of the Division to the Dividend.

Upon these and some such other Principles is grounded all that Doctor Wallis has said not onely in his *Elenchus* of my Geometry, but also in his Treatises of the Angle of Contact, and in his *Arithmetica Infinitorum*; which two last I have here in two or three leaves wholly and cleerly confuted. And I verily believe that since the beginning of the world there has not been, nor ever shall be so much absurdity written in Geometry, as is to be found in those books of his; with which there is so much presumption joyned, that an *ἀποκατάστασις* of the like conjunction cannot be expected in less then a Platonick year. The cause whereof I imagine to be this, that he mistook the study of *Symboles* for the study of Geometry, and thought *Symbolicall* writing to be a new kinde of Method, and other mens Demonstrations set down in *Symboles* new Demonstrations. The way of *Analysis* by Squares, Cubes, &c. is very ancient, and usefull for the finding out whatsoever is contained in the nature and generation of rectangled Plains (which also may be found without it) and was at the highest in *Vieta*; but I never saw any thing added thereby to the Science of Geometry, as being a way wherein men go round from the Equality of rectangled Plains to the Equality of Proportion, and thence again to the Equality of rectangled Plains; wherein the *Symboles* serve only to make men go faster about, as greater Winde to a Winde-mill.

It is in Sciences as in Plants; Growth and Branching is but the Generation of the Root continued; nor is the Invention of Theoremes any thing else but the knowledge of the Construction of the Subject prosecuted. The unsoundness of the Branches are no prejudice to the Roots; nor the Faults of Theoremes to the Principles. And active Principles will correct false Theoremes

The Epistle Dedicatory.

remes if the Reasoning be good ; but no Logique in the world is good enough to draw evidence out of false or unactive Principles. But I detain your Lordship too long. For all this will be much more manifest in the following Discourses: wherein I have not onely explained and rectified many of the most important Principles of Geometry : but also by the examples of those errors which have been committed by my Reprehenders, made manifest the evil Consequence of the Principles they now proceed on. So that it is not only my own Defence that I here bring before you, but also a positive doctrine concerning the true Grounds, or rather Atomes of Geometry : which I dare only say are very singular: but whether they be very good or not, I submit to your Lordships judgement. And seeing you have been pleased to bestow so much time (with great success) in the reading of what has been written by other men in all kindes of Learning. I humbly pray your Lordship to bestow also a little time upon the reading of these few and short Lessons ; and if your Lordship finde them agreeable to your Reason and Judgement, let me (notwithstanding the clamour of my Adversaries) be continued in your good opinion, and still retain the honour of being

My most Noble Lord,
London, *June 10.*
1656.

Your Lordships most
Humble and Obliged
Servant.
THOMAS HOBBS.

ERRATA.

Page 2. l. 11. for *Art*, r. *act*, l. 12. for *ἐπαγωγὴς*, r. *ἐπαγωγισ*, p. 6. l. 23. for *Mr.* r. *Sir*, p. 21.
l. 22. for *Proposition*, r. *Proportion*, p. 55. l. 20. for *Senſtoribus*, r. *Senatoribus*, p. 56. l. 8. for
Serherim, r. *Serberius*.

remains if the Reasoning be not
 good enough to justify the
 ples. But I desire your Lordship to be
 much more in the following Disquisition
 not only examined and treated many of the
 Principles of Geometry; but also by the
 rors which have been committed by my Lordship
 manifest the evil Consequence of the Principles
 ceded on. So that it is not only my own Disposition
 before you, but also a positive duty to
 Grounds, or rather Arguments of Geometry; and I
 say are very singular; but whether they be
 submitted to your Lordship's judgement. And being
 pleased to bestow so much time (with great industry) in the
 ing of what has been written by others in all kinds of
 ing. I humbly pray your Lordship to bestow this
 upon the reading of these few and short Letters, and if
 I should find them agreeable to your Reason and Judgment
 term (notwithstanding the nature of my Affairs) to con-
 tinue in your good opinion; and still retain the honour of being

London, Jan. 10.
 1676.
 Your Lordship's most
 Humble and Obedient
 Servant,
 Thomas Hobbes

ERRATA

Page 1. Line 1. for Proposition 1. read Proposition 2.
 Page 2. Line 1. for Proposition 2. read Proposition 1.
 Page 3. Line 1. for Proposition 3. read Proposition 4.
 Page 4. Line 1. for Proposition 4. read Proposition 3.

LESSONS

OF THE

PRINCIPLES

OF

Geometry, &c.

To the egregious Professors of the Mathematicks, one of Geometry, the other of Astronomy, in the Chairs set up by the Noble and Learned Sir *Henry Savile*, in the University of *Oxford*.

LESSON I.



Suppose (most egregious Professors) you know already that by Geometry (though the word import no more but the measuring of Land) is understood no less the measuring of all other Quantity, then that of Bodies. And though the Definition of Geometry serve not for proof, nor enter into any Geometrical Demonstration, yet for understanding of the Principles of the Science, and for a Rule to judge by, who is a Geometrician, and who is not, I hold it necessary to begin therewith.

Geometry is the Science of determining the quantity of any thing, not measured, by comparing it with some other Quantity, or Quantities measured. Which Science therefore whosoever shall go about to teach, must first be able to tell his Disciple what Measuring or Dimension is; by what each several kind of Quantity is Measured; what Quantity is, & what are the several kinds thereof. Therefore as they who handle any one part of Geometry, determine by Definition the signifie

cation of every word which they make the Subject, or Prædicate of any Theoreme they undertake to demonstrate; so must he which intendeth to write a whole body of Geometry, Define and Determine the meaning of whatsoever word belongeth to the whole Science; The design of *Euclid* was to demonstrate the Properties of the five regular bodies mentioned by *Plato*; in which Demonstrations there was no need to alledge for Argument the Definition of Quantity, which it may be was the cause he hath not any where Defined it, but done what he undertook without it. And though having perpetually occasion to speak of measure, he hath not Defined Measure; yet instead thereof he hath in the beginning of his first Element, assumed an Axiome which serveth his turn sufficiently, as to the measure of lines, which is the eighth Axiome; That those things which lye upon one another all the way (called by him *σφαμίζοντα*) are equall. Which Axiome is nothing else but a description of the Art of Measuring Length, and Superficies. For this *σφαμίζοντα* can have no place in solid bodies, unless two bodies could at the same time be in one place. But amongst the Principles of Geometry universall, the Definitions are necessary, both of Quantity and Dimension.

Quantity is that which is signified by what we answer to him that asketh, *How much* any thing is, and thereby determine the magnitude thereof. For magnitude being a word indefinite, if a man ask of a thing *Quantum est*, that is, *How much it is*; we do not satisfy him by saying it is magnitude or quantity, but by saying it is *Tantum, so much*. And they that first called it in Greek *πληρότης*, and in Latine *Quantity*, might more properly have called it, in Latine *Tantity*, and in Greek *πληρότης*; and we, if we allowed our selves the Eloquence of the Greeks and Latines, should call it the *So-muchness*.

There is therefore to every thing concerning which a man may ask without absurdity *how much it is*, a certain Quantity belonging; determining the magnitude to be *so much*. Also wheresoever there is *more* and *less*, there is one kinde of Quantity or other. And first, there is the Quantity of Bodies, and that of three kinds; Length, which is by one way of Measuring; Superficies made of the complication of two Lengths, or the Measure taken two wayes; and Solid which is the complication of three lengths, or of the measure taken three wayes; for breadth or thickness are but other Lengths. And the Science of Geometry so far forth as it contemplateth Bodies onely, is no more but by Measuring the length of one or more lines, and by the position of others known in one and the same Figure, to Determine by ratiocinations, how much is the Superficies; and by Measuring Length, Breadth, and Thickness, to determine the Quantity of the whole Body. Of this kinde of Magnitudes and Quantities the Subject is Body.

And because for the computing of the Magnitudes of Bodies, it is not necessary that the Bodies themselves should be present (the Ideas and memory of them supplying their presence) we reckon upon those Imaginary Bodies, which are the Quantities themselves, and say the length is so great, the breadth so great, &c. which in truth is no better then to say the length is so long, or the breadth so broad, &c. But in the mind of an intelligent man it breedeth no mistake.

Besides the Quantity of Bodies, there is a Quantity of Time. For seeing men, without absurdity do ask how much it is; by answering *Tantum, so much*, they make manifest there is a quantity that belongeth unto Time, namely, a Length. And because Length cannot be an accident of Time, which is it self an accident, it is the accident of a Body; and consequently the length of the Time, is the Length of the Body; by which Length or Line, we determine how much the Time is, supposing some Body to be moved over it.

Also we not improperly ask with *how much* Swiftnes a Body is moved, and consequently there is a Quantity of Motion or Swiftnes, and the measure of that Quantity is also a line. But then again, we must suppose another motion, which determineth the time of the former. Also of Force there is a Question of *How much*, which is to be answered by *so much*; that is, by Quantity. If the Force consist in Swiftnes, the Determination is the same with that of Swiftnes, namely, by a Line; if in Swiftnes and Quantity of Body jointly, then by a Line and a Solid; or if in quantity of Body onely (as Weight) by a Solid onely.

So

So also is Number Quantity; but in no other sense than as a line is Quantity divided into equal parts.

Of an Angle, which is of two Lines whatsoever they be meeting in one point the digression or opening in other points, it may be asked how great is that digression? This Question is answered also by Quantity. An Angle therefore hath Quantity, though it be not the subject of Quantity; for the body only can be the subject, in which Body whole straight line are marked.

And because two lines may be made to divaricate by two causes; one, when having one end common, and immovable, they depart one from another at the other ends circularly, and this is called simply an Angle; and the Quantity thereof is the Quantity of the Arch, which the two lines intercept.

The other cause is the bending of a straight line into a circular or other crooked line, till it touch the place of the same line, whilst it was straight, in one only point. And this is called an Angle of contingence. And because the more it is bent, the more it digresseth from the Tangent, it may be asked how much more? and therefore the answer must be made by Quantity; and consequently an Angle of Contingence hath its Quantity as well as that which is called simply an Angle. And in case the digression of two such crooked lines from the Tangent be uniform as in Circles, the Quantity of their digression may be determined. For if a straight line be drawn from the point of Contact, the digression of the lesser Circle will be to the digression of the greater Circle, as the part of the line drawn from the point of Contact, and intercepted by the Circumference of the greater Circle is to the part of the same line intercepted by the Circumference of the lesser Circle, or, which is all one, as the greater Radius is to the lesser Radius. You may guess by this what will become of that part of your last Book, where you handle the Question of the Quantity of an Angle of Contingence.

Also there lyeth a Question of how much Comparatively, one magnitude is to another magnitude, as how much water is in a Tun in respect of the Ocean, how much in respect of a Pin; little in the first respect, much in the latter. Therefore the Answer must be made by some respective Quantity. This respective Quantity is called Ratio and Proportion, and is determined by the Quantity of their differences; and if their differences be compared also with the Quantities themselves that differ, it is called simply Proportion, or Proportion Geometrically. But if the differences be not so compared, then it is called Proportion Arithmetically. And where the difference is none, there is no Quantity of the Proportion, which in this case is but a bare comparison.

Also concerning Heat, Light, and divers other Qualities, which have degrees, there lyeth a question of how much, to be answered by a so much, and consequently they have their Quantities, though the same with the Quantity of Swiftness: because the intensions and remissions of such Qualities are but the intensions and remissions of the Swiftness of that motion by which the Agent produceth such a quality. And as Quantity may be considered in all the operations of Nature, so also doth Geometry run quite thorow the whole body of Naturall Philosophy.

To the Principles of Geometry the definition appertaineth also of a Measure, which is this, One Quantity is the Measure of another Quantity, when it, or the Multiple of it, is Coincident in all points, with the other Quantity. In which Definition in stead of that *ἰσότης* 2d of Euclid, I put Coincidence. For that superposition of Quantities by which they render the word *ἰσότης* cannot be understood of Bodies, but only of Lines and Superficies. Nevertheless many Bodies may be Coincident successively with one and the same place, and that place will be their Measure, as we see practised continually in the measuring of Liquid Bodies, which Art of Measuring may properly be called *ἰσμετρούς*, but not Superposition.

Also the definitions of Greater, Less, and Equal, are necessary Principles of Geometry. For it cannot be imagined that any Geometrician should demonstrate to us the Equality, and Inequality.

quality of magnitudes, except he tell us first what those words do signifie. And it is a wonder to me, that *Euclide* hath not any where defined what are Equals, or at least, what are Equall Bodies, but serveth his turn throughout with that forementioned *ἐπαγόμεναι*, which hath no place in Solids, nor in Time, nor in Swiftnes, nor in Circular, or other crooked lines; and therefore no wonder to me, why Geometry hath not proceeded to the calculation neither of crooked lines, nor sufficiently of Motion, nor of many other things, that have proportion to one another.

Equall Bodies, Superficies, and Lines are those of which every one is capable of being co-incident with the place of every one of the rest. And Equall Times wherein with one and the same Motion Equall lines are described. And Equally Swift are those Motions by which we run over equal spaces in any time determined by any other motion. And universally all Quantities are Equall, that are measured by the same number of the same Measures.

It is necessary also to the Science of Geometry, to define what Quantities are of one and the same kind, which they call *Homogeneous*; the want of which definitions hath produced those wranglings (which your Book *De Angulo Contactus* will not make to cease) about the Angle of Contingence.

Homogeneous quantities are those which may be compared by (*ἐπαγόμεναι*) application of their Measures to one another; So that Solids and Superficies are Heterogeneous quantities, because there is no coincidence or application of those two dimensions.

No more is there of Line and Superficies, nor of Line and Solid, which are therefore Heterogeneous. But Lines and Lines, Superficies and Superficies, Solids and Solids, are Homogeneous.

Homogeneous also are Line, and the Quantity of Time; because the Quantity of Time is measured by application of a Line to a Line; for though Time be no Line, yet the Quantity of Time is a Line, and the length of two Times is compared by the length of two Lines.

Weight, and Solid have their Quantity Homogeneous, because they measure one another by application, to the beam of a Balance. Line and Angle simply so called have their Quantity Homogeneous, because their measure is an Arch or Arches of a Circle applicable in every point to one another.

The Quantity of an Angle simply so called, and the Quantity of an Angle of Contingence are Heterogeneous. For the measures by which two Angles simply so called are compared, are in two coincident Arches of the same Circle; but the measure by which an Angle of Contingence is measured, is a straight line intercepted between the point of Contact, and the Circumference of the Circle; and therefore one of them is not applicable to the other; and consequently of these two sorts of Angles, the Quantities are Heterogeneous. The Quantities of two Angles of Contingence are Homogeneous; for they may be measured by the *ἐπαγόμεναι* of two Lines, whereof one extreame is common, namely, the point of Contact, the other Extreames, are in the Arches of the two Circles.

Besides this knowledge of what is Quantity and Measure, and their severall sorts, it becometh a Geometrician to know why, and of what they are called Principles. For not every Proposition that is evident, is therefore a Principle. A Principle is the beginning of something. And because Definitions are the beginnings or first Propositions of Demonstration, they are therefore called Principles, Principles, I say, of Demonstration. But there be also necessary to the teaching of Geometry other Principles, which are not the beginnings of Demonstration, but of Construction, commonly called Petitions; as that it may be granted that a man can draw a straight Line, and produce it; and with any Radius; on any Center describe a Circle, and the like. For that a man may be able to describe a square, he must first be able to draw a straight line; and before he can describe an *Æquilaterall Triangle*, he must be able first to describe a Circle. And these Petitions are therefore properly called Principles (not of Demonstration, but) of Operation. As for the commonly received third sort of Principles, called *Common Notions*, they are Principles, onely by permission of him that is the Disciple; who being *ῥησιμους*, and comming not to cavill but to learn, is content to receive them (though Demonstrable)

Demonstrable) without their Demonstrations. And though Definitions be the only Principles of Demonstration, yet it is not true that every Definition is a Principle. For a man may so precisely determine the signification of a word, as not to be mistaken, yet may his Definition be such, as shall never serve for proof of any Theorem, nor ever enter into any demonstration (such as are some of the Definitions of *Euclid*) and consequently can be no beginnings of Demonstration, that is to say, no Principles.

All that hitherto hath been said, is so plain and easie to be understood, that you cannot (most Egregious Professors) without discovering your ignorance to all men of reason, though no Geometricians, deny it. And the same (saying that the words are all to be found in Dictionaries) new; also to him that means to learn, not only the Practice, but also the Science of Geometry necessary, and (though it grieve you) mine. And now I come to the Definitions of *Euclid*.

The first is of a Point. *Σημείον &c. Signum est, cujus est pars nulla*, that is to say, a Marke is that of which there is no part. Which definition, not onely to a candid, but also to a rigid construer is sound and usefull. But to one that neither will interpret candidly, nor can interpret accurately, is neither usefull nor true. Theologers say the Soul hath no part, and that an Angel hath no part, yet do not think that Soul or Angel is a point. A mark, or as some put in stead of it *σημα*, which is a mark with a hot Iron, is visible; if visible then it hath Quantity, and consequently may be divided into parts innumerable. That which is indivisible is no Quantity; and if a point be not Quantity, seeing it is neither Substance nor Quality, it is nothing. And if *Euclid* had meant it so in his definition, (as you pretend he did) he might have defined it more briefly, (but ridiculously) thus, a Point is nothing. Sir Henry Savile was better pleased with the Candid interpretation of *Proclus*, that would have it understood respectively to the matter of Geometry. But what meaneth this respectively to the matter of Geometry? It meaneth this, that no Argument in any Geometricall demonstration should be taken from the Division, Quantity, or any part of a Point; which is as much as to say, a Point is that whose Quantity is not drawn into the demonstration of any Geometricall conclusion; or (which is all one) whose Quantity is not considered.

An accurate interpreter might make good the definition thus, a Point is that which is undivided; and this is properly the same with *cujus non est pars*: for there is a great difference between undivided, and indivisible, that is, between *cujus non est pars*, and *cujus non potest esse pars*. Division is an Act of the understanding; the understanding therefore is that which maketh parts, and there is no part where there is no consideration but of one. And consequently *Euclid's* definition of a Point, is accurately true, and the same with mine, which is, that a Point is that Body whose Quantity is not considered. And considered, is that (as I have defined it, Chap. 1. at the end of the third Article) which is not put to account in demonstration.

Euclid therefore seemeth not to be of your opinion, that say a Point is nothing. But why then doth he never use this definition in the Demonstration of any Proposition? Whether he useth it expressly or no, I remember not; but the 16th. Proposition of the third Book without the force of this definition is undemonstrated.

The second Definition is of a Line. *γραμμή δὲ μήν ἐστί ἀπλάνη. Linea est longitudo latitudinis expertis*, a Line is length which hath no breadth; and it candidly interpreted, sound enough, though rigorously, not so. For to what purpose is it to say Length not Bread, when there is no such thing as a broad length. One Path may be broader then another Path, but not one Mile then another Mile; and it is not the Path but the Mile which is the ways Length. If therefore a man have any ingenuity he will understand it thus, That a Line is a Body whose length is considered without its breadth, else we must say absurdly a broad length; or untruly, that there be bodies which have length and yet no breadth; and this is the very sense which *Apollonius* (saith *Proclus*) makes of this Definition; when we measure, sayes he, the length of a way, we take not in the breadth or depth, but consider onely one Dimension. See this of *Proclus*.

Proclus cited by Sir Henry Savile, where you shall finde the very word temper.

The fourth Definition is of a straight line, thus *Εὐθεία ἑστὶν ἡ ἐν ἑαυτῇ ὅμοιως ἐκτεταμένη, ὅρα καὶ ἡ ἀπὸ τοῦ αὐτοῦ σημείου ἐκτεταμένη.* A straight line is that which lies equally (or perhaps evenly) between its own Points. This Definition is inexcusable. Between what Points of its own can a straight line lye but between its extremes? And how lies it evenly between them, unless it swerve no more from some other line which hath the same Extreams, one way then another? And then why are not between the same Points both the lines straight? How bitterly, and with what insipide jests, would you have reviled *Euclide* for this. If living now he had written a *Leviathan*. And yet there is somewhat in this Definition to help a man, not onely to conceive the nature of a straight line (for who doth not conceive it?), but also to express it. For he meant perhaps to call a straight line that which is all the way from one Extream to another, equally distant from any two or more such Lines as being like and equall have the same Extreams. So the Axis of the Earth is all the way equally distant from the circumference of any two or more Meridians. But then before he had defined a straight line, he should have defined what lines are like, and what are equall. But it had been best of all, first to have defined crooked lines, by the possibility of a deduction or setting further asunder of their extremes; and then straight Lines, by the impossibility of the same.

The seventh Definition, which is that of a plain Superficies, hath the same fault.

The Eighth is of a plain Angle. *Ἐν τῷ αὐτῷ ὁρίῳ ἑστὶν ἡ ἐν ἑαυτῇ ὁμοιότης, ὅρα καὶ ἡ ἀπὸ τοῦ αὐτοῦ σημείου ἐκτεταμένη.* A plain Angle is the inclination one towards another of two lines that touch one another in the same Plain, and lye not in the same straight line. Besides the faults here observed by Mr. Henry Savile, as the clause of not lying in the same straight line, and the obscurity or equivocation of the word *inclination*, there is yet another, which is, that by this Definition two Right Angles together taken, are no Angle; which is a fault which you somewhere (asking leave to use the word *Angle καταχρηστικῶς*, acknowledge, but avoid not. For in Geometry, where you confesse there is required all possible accurateness, every *καταχρηστικῶς* is a fault. Besides you see by this Definition, that *Euclide* is not of your, but of *Clavius* his opinion. For it is manifest that the two lines which contain an Angle of Contact, incline one towards another, and come together in a Point, and lye not in the same straight line, and consequently make an Angle.

The thirteenth Definition is exact, but makes against your Doctrine, that a Point is nothing. Examine it. *Ὁ ὅρος ἐστὶν ὁ τέρμας ἐστὶν ὁ ὅρος.* Terminus est quod aliquid extremum est. A Term or Bound is that which is the Extream of any thing. We had before, The extremes of a Line are Points. But what is in a Line the extream, but the first or last part, though you may make that part as small as you will? A point is therefore a part, and nothing is no extream.

The fourteenth, *Ἐν τῷ αὐτῷ ὁρίῳ ἑστὶν ἡ ἐν ἑαυτῇ ὁμοιότης, ὅρα καὶ ἡ ἀπὸ τοῦ αὐτοῦ σημείου ἐκτεταμένη.* Figura est (subaudi Quantitas) quae ab aliquo, vel aliquibus terminis undiq; continetur sive clauditur. A Figure is Quantity contained within some bound or bounds. Or shortly thus, A Figure is Quantity every way determined, is in my opinion as exact a Definition of a Figure as can possibly be given, though it must not be so in yours. For this determination is the same thing with circumscription; and whatsoever is any where (*ubi unq;*) definitive is there also circumscribed; and by this means, the distinction is lost, by which Theologers, when they deny God to be in any place, save themselves from being accused of saying he is no where; for that which is no where is nothing. This definition of *Euclide* cannot therefore possibly be embraced by you that carry double, namely, *Mathematicks* and *Theology*. For if you reject it, you will be cast out of all Mathematick Schooles; and if you maintain it, from the Society of all School-Divines, and lose the thanks of the Favour you have shewn (you the Astronomer) to Bishop Bramhall.

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The fifteenth is of a Circle. *Κύκλος ἐστὶν ὅταν ἐκείνῳ, &c.* A Circle is a plain Figure comprehended by one line which is called the Circumference, to which Circumference all the straight lines drawn from one of the points within the Figure, are equal to one another. This is true. But if a man had never seen the generation of a Circle by the motion of a Compass or other equivalent means, it would have been hard to persuade him, that there was any such Figure possible. It had been therefore not amiss first to have let him see that such a Figure might be described. Therefore so much of Geometry is no part of Philosophy, which seeketh the proper passions of all things in the generation of the things themselves.

After the fifteenth till the last or thirty fifth Definition all are most accurate, but the last, which is this, *Parallel straight lines are those which being in the same Plane, though infinitely produced both wayes shall never meet.* Which is lesse accurate. For how shall a man know that there be straight lines, which shall never meet, though both wayes infinitely produced? Or how is the Definition of Parallels, that is, of lines perpetually equidistant good, wherein the nature of equidistance is not signified? Or if it were signified, why should it not comprehend, as well the Parallelism of Circular and other crooked lines, as of straight, and as well of Superficies, as of lines? By Parallels is meant equidistant both Lines and Superficies, and the word is therefore not well defined without defining first equality of distance. And because the distance between two lines or Superficies, is the shortest line that can joyn them, there either ought to be in the definition, the shortest distance, which is that of the Perpendicular, and without inclination, or the distance in equall inclination, that is, in equall Angles. Therefore if Parallels be defined to be those Lines or Superficies, where the Lines drawn from one to an other in equall Angles be equal, the Definition, as to like Lines, or like Superficies, will be Universal and Convertible. And if we add to this Definition, that the equall Angles be drawn not opposite wayes, it will be absolute, and Convertible in all Lines and Superficies; and the definition will be this, *Parallels are those Lines and Superficies between which every line drawn, in any Angle, is equall to any other line drawn in the same Angle the same way.* For by this Definition the distance between them will perpetually be equall, and consequently they will never come nearer together, how much, or which way soever they be produced. And the converse of it will be also true, *If two Lines, or two Superficies be Parallel, and a straight line be drawn from one to the other, any other straight lines drawn from one to the other in the same Angle, and the same way, will be equall to it.* This is manifestly true, and (most egregious Professors) new, at least to you.

And thus much for the Definitions placed before the first of Euclides Elements.

Before the third of his Elements is this Definition. *In circulo aequaliter distare à centro rectæ lineæ dicuntur, cum perpendiculares quæ à centro in ipsas ducuntur sunt æquales.* In a circle two straight Lines are said to be equally distant from the Center, upon which the perpendiculars drawn from the Center are equal. This is true; but it is rather an Axiome than a Definition, as being demonstrable that the Perpendicular is the measure of the distance between a Point and a straight or crooked Line.

Before the fifth Element the first Definition is of a Part. *Pars est magnitudo magnitudinis, minor majori, cum minor metitur majorem.* A Part is one magnitude of another, the lesse of the greater, when the lesse measureth the greater. From which Definition it followeth, that more then the half is not a part of the whole. But because Euclide meaneth here an Aliquot part, as a half, a third, or a fourth, &c. It may pass for the Definition of a measure under the name of part; as thus, *a Measure is a Part of the whole, when multiplied, it may be equall to the whole,* though properly a Measure is external to the thing measured, and not the Aliquot part it self; but equall to an Aliquot part.

But the third Definition is intollerable. It is the definition of λόγος in Latin Ratio, in English Proportion; in this manner, *λόγος ἐστὶ δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πλάκην τὰ πρὸς ἀλλήλα ποία ἔσσι.* Ratio est duarum magnitudinum ejusdem generis mutua quædam secundum quantitatem habitudo. Proportion is a certain mutuall habitude in Quantity, of two magnitudes.

nitules of the same kind, one to another. First, we have here *ignotum per ignotum*; for every man understandeth better what is meant by *Proportion* than by *Habitude*. But it was the phrase of the Greeks when they named like Proportions, to say, the first to the second $\delta\upsilon\tau\omega\varsigma\ \acute{\epsilon}\chi\epsilon\iota$, *id est, ita se habet*, and in English, *is as*, the third to the fourth. As for Example (in the Proportions of two to four, and three to six) to say two to four $\delta\upsilon\tau\omega\varsigma\ \acute{\epsilon}\chi\epsilon\iota$, *id est, ita se habet*, *id est, is as*, three to six. From which phrase *Euclide* made this his Definition of Proportion by $\pi\omicron\iota\alpha\ \acute{\alpha}\nu\iota\sigma\iota\varsigma$, which the Latines translate *quedam habitudo*. *Quadam* in a Definition is a most certain note of not understanding the word defined; And in Greek $\pi\omicron\iota\alpha\ \acute{\alpha}\nu\iota\sigma\iota\varsigma$ is much worse; for to render rightly the Greek definition, we are to say in English, that Proportion is a what-qual-I-call-it *isness*, or *soness* of two magnitudes &c. Then which nothing can be more unworthy of *Euclide*. It is as bad as any thing was ever said in Geometry by *Orontius*, or by *Dr. Wallis*. That Proportion is Quantity compared, that is to say, little or great in respect of some other Quantity (as I have above defined it) is I think intelligible.

The fourth is, $\text{Ἀναλογία δὲ εἰνὴ τῶν λόγων ὁμοίότης}$. *Proportio verò est rationum similitudo*. Here we have no one word by which to render Ἀναλογία ; for our word *Proportion*, is already bestowed upon the rendering of λόγος . Nevertheless the Greek may be translated into English thus, *Iterated Proportion is the similitude of Proportions*. But *Iterated Proportion* is the same with *eadem Ratio*. To what purpose then serveth the sixth Definition, which is of *eadem Ratio*? For Ἀναλογία and *eadem Ratio*, and *Similitudo Rationum*, are the same thing, as appeareth by *Euclide* himself; where he defines those Quantities, that are in the same Proportion by ἀνάλογον . Therefore the sixth Definition is but a *Lemma*, and assumed without demonstration.

The fourteenth, *Compositio Rationis est sumptio Antecedentis cum Consequente, seu unius, ad ipsum consequentem*. To Compound Proportions, is to take both Antecedent, and Consequent together, as one magnitude, and compare it to the consequent; Is good; though he might have compared it as well with the Antecedent; For both ways it had been a Composition of Proportions. We are to note here, that the Composition defined in this place by *Euclide*, is not adding together of Proportions, but of two Quantities that have Proportion. And therefore it is not the same Composition which he defineth in the fourth place before the sixth Element; for there

He defineth the addition of one Proportion to another Proportion in this manner $\text{λόγος ἐκ λόγων συνησθῆναι λέγεται}$, &c. A Proportion is said to be compounded of Proportions, when their Quantities multiplye into one another make a Proportion; as when we would compound, or add together, the Proportions, of three to two, and of four to five, we must multiply three and four, which maketh twelve, and two and five which maketh ten. And then the Proportion of twelve to ten, is the sum of the Proportions, of three to two, and of four to five; which is true; but not a definition; for it may and ought to be demonstrated. For to define what is addition of two Proportions (which are always in four Quantities, though sometimes one of them be twice named), we are to say, that they are then added together when we make the second to another in the same Proportion, which the third hath to the fourth.

And thus much of the Definitions; of which, some, very few, you see are faulty; the rest either accurate, or good enough if well interpreted. For before the rest of the Elements all are accurate, notwithstanding, that you allow not for good any definition in Geometry that hath in it the word *motion*: of which there be divers before the Eleventh Element. But I must here put you in minde, that Geometry being a Science, and all Science proceeding from a pre cognition of causes, the definition of a Sphere, and also of a Circle, by the generation of it, that is to say, by motion, is better then by the equality of distance from a Point within.

The second sort of Principles, are those of Construction, usually called *Postulates*, or *Petitions*. For as for those *notiones communes*, called *Axiomes*, they are from the definitions of their terms demonstrable, though they be so evident as they need not demonstration. These Petitions are by *Euclide* called *Αἰτήματα*, such as are granted by favour, that is, simply Petitions whereas by *Axiome* is understood that which is claimed as due. So that between *Αἰτήματα* and *Αἰτήματα* there is this other difference, that this later is simply a Petition, the former a Petition of Right.

Of Petitions simply, the first is, *That from any Point to any Point may be drawn a straight Line.* The second, *That a finite straight Line may be produced.* The third, *That upon any Center, at any distance may be described a Circle.* All which are both evident, and necessary to be granted.

And by all these a man may easily perceive that *Euclide* in the definitions of a Point, a Line, and a Superficies, did not intend that a Point should be Nothing, or a Line be without Latitude, or a Superficies without Thickness; for if he did, his Petitions are not only unreasonable to be granted, but also impossible to be performed. For Lines are not drawn but by Motion; and Motion is of Body only. And therefore his meaning was, that the Quantity of a Point, the Breadth of a Line, and the Thickness of a Superficies were not to be considered, that is to say, not to be reckoned in the demonstration of any Theoreme concerning the Quantity of Bodies, either in Length, Superficies, or Solid.

Of the Faults that Occurre in Demonstration.

To the same egregious Professors of the Mathematicks in the University of Oxford.

LESSON II.

THERE be but two causes from which can spring an error in the demonstration of any conclusion in any Science whatsoever. And those are Ignorance or want of understanding, & Negligence. For as in the adding together of many and great Numbers, he cannot fail, that knoweth the Rules of Addition, and is also all the way so careful, as not to mistake one number, or one place for another; so in any other Science, he that is perfect in the Rules of Logick, and is so watchfull over his Pen, as not to put one word for another, can never fail of making a true, though not perhaps the shortest and easiest demonstration.

The Rules of Demonstration are but of two kinds; One, that the Principles be true and evident Definitions; the other, that the Inferences be necessary. And of true and evident definitions, the best are those which declare the cause or generation of that Subject, whereof the proper passions are to be demonstrated. For Science is that knowledge which is derived from the comprehension of the cause. But when the cause appeareth not, then may, or rather must we define some known property of the Subject, and from thence derive some possible way, or wayes of the
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generation. And the more quick of generation are explained, the more easy will be the derivation of the Properties; whereof some are more immediate to one, some to another generation. He therefore that proceedeth from untrue, or not understood definitions, is ignorant of that he goes about; which is an ill-favoured fault, be the matter he undertaketh easy or difficult; because he was not forced to undergo a greater change then he could carry through. But he that from right definitions maketh a false conclusion, erreth through humane frailty, as being less awake, more troubled with other thoughts, or more in haste when he was in writing. Such faults, unless they be very frequent, are not attended with shame, as being common to all men, or are at least less ugly then the former; except then, when he that committeth them reprehendeth the same in other men. For that is in every man intolerable, which he cannot tolerate in another. But to the end that the faults of both kinds may by every man be well understood, it will not be amiss to examine them by some such Demonstrations, as are publicly extant. And for this purpose I will take such as are in mine and in your Books, and begin with your *Elementus* of the Geometry contained in my Book *de Corpore*; to which I will also joyn your Book lately set forth concerning the *Angle of Contact*, *Conique sections*, and your *Arithmetica Infinitorum*; and then examine the rest of my Philosophy, and yours that oppose it. For I will take leave to consider you both every where as one Author, because you publicly declare your approbation of one anothers doctrine.

My first Definition is of a Line, of Length, and of a Point; *The way* (say I) of a Body moved, in which magnitude (though it alwayes have some magnitude) is not considered, is called a Line; and the space gone over by that motion, Length, or one and a simple Dimension. To this Definition you say, First, what Mathematician did ever thus define a Line or Length? Whether you call in others for help or testimony, it is not done like a Geometrician; for they use not to prove their conclusions by witnesses, but relye upon the strength of their own reason; and when your witnesses appear, they will not take your part. Secondly, you grant that what I say is true, but not a Definition. But to tell you truly what it is which we call a Line, is to define a Line. Why then is not this a Definition? Because (say you in the first place) it is not a reciprocal proposition. But by your favour it is reciprocal. For not only the way of a Body whose Quantity is not considered, is a Line; but also every Line is (or may be conceived to be) the way of a Body so moved. And if you object that there is a difference between *is* and *may be conceived to be*, *Euclide* whom you call to your aid, will be against you in the fourteenth Definition before his eleventh Element; Where he defines a Sphere just as convertibly as I define a Line; except you think the Globes of the Sun and Stars cannot be Globes, unless they were made by the circumduction of a Semicircle; And again in the eighteenth definition, which is of a Cone, unless you admit no Figure for a Cone, which is not generated by the Revolution of a Triangle; And again, in the twentieth Definition, which is of a Cylinder, except it be generated by the circumvolution of a Parallelogram. *Euclide* saw that what proper passion soever should be derived from these his Definitions, would be true of any other Cylinder, Sphere, or Cone, though it were otherwise generated; And the description of the generation of any one being by the imagination applicable to all (which is equivalent to convertible) he did not believe that any rational man could be misled by learning Logick, to be offended with it. Therefore this exception proceedeth from want of understanding, that is, from ignorance of the nature, and use of a Definition.

Again, you object and ask *What need is there of motion, or of Body moved, to make a man understand what is a Line? Are not Lines in a Body at rest, as well as in a Body moved? And is not the distance of two resting points Length, as well as the measure of the passage? Is not Length one and a simple dimension, and one and a simple dimension Line? Why then is not Line and Length all one? See how impertinent these questions are.* *Euclide* defines a Sphere to be a Solid Figure described by the revolution of a Semicircle, about the unmoved Diameter. *Why do you not ask what need there is to the understanding of what a Sphere is, to bring in the motion of a Semicircle? Is not a Sphere to be understood without such motion? Is not the*

the Figure so made, a Sphere without this motion? And where he defines the Axis of a Sphere to be that unmoved Diameter, may not you ask, whether there be no Axis of a Sphere, when the whole Sphere, Diameter and all is in motion? But it is not my purpose to defend my definition by the example that of *Euclide*. Therefore first, I say, to me howsoever it may be to others, it was fit to define a Line by Motion. For the generation of a Line is the Motion that describes it. And having defined Philosophy in the beginning, to be the knowledge of the properties from the generation, it was fit to define it by its generation. And to your question, *Is not distance Length?* I answer, that though sometimes distance be æquivalent to Length; yet certainly the distance between the two ends of a thread wound up into a Clew is not the length of the thread; for the length of the thread is equall to all the windings whereof the Clew is made. But if you will needs have distance and length to be all one, tell me of what, the distance between any two Points is the Length. Is it not the length of the way? And how is that called Way, which is not defined by some motion? And have not severall wayes between the same places, as by Land and by Water, severall lengths? But they have but one distance, because the distance is the shortest way. Therefore between the length of the Path, and the distance of the Places, there is a real difference in this case, and in all cases a difference of the consideration. Your objection, that Line is Longitude, proceeds from want of understanding English. Do men ever ask what is the Line of a thread, or the Line of a Table, or of any other Body? Do they not alwayes ask what is the length of it? And why, but because they use their own judgements, nor yet corrupted by the subtilty of mistaken Professors. *Euclide* defines a Line, to be length without breadth. If those terms be all one, why said he not that a Line, is a Line without breadth? But what Definition of a Line give you? None. Be contented then with such as you receive, and with this of mine, which you shall presently see is not amiss.

Your next objections are to my Definition of a Point. Which Definition adhereth to the former in these words, *and the Body its self is called a Point*. Here again you call for help; *Quis unquam mortalium, &c. What mortall man, what sober man did ever so Define a Point?* 'Tis well, and I take it to be an honour to be the first that do so. But what objection do you bring against it? This, that a point added to a Point (if it have magnitude) makes it greater. I lay it doth so, but then presently it loseth the name of a Point, which name was given to signifie that it was not the meaning of him that us'd it in demonstration, to add, subtract, multiply, divide, or any way compute it. Then you come in with *perhaps you will say though it have magnitude, that magnitude is not considered*. You need not say *perhaps*. You know I affirm it; and therefore your Argument might have been left out, but that it gave you occasion of a digression into lewric language.

And whereas you ask why I defined not a Point thus, *Punctum est corpus quod non consideratur esse corpus, & magnum quod non consideratur esse magnum*. I will tell you why. First, because it is not Latine. Secondly, because when I had defined it by *corpus*, there was no need to Define it again by *magnum*. I understand very well this language, *Punctum est corpus, quod non consideratur ut corpus*. A Point is a Body not considered as Body. But *Punctum est corpus, quod non consideratur esse corpus, vel esse magnum*, is not Latine, nor the version of it, *A Point is a Body which is not considered to be a Body*, English. My Definition was, that a Point is that Body whose magnitude is not considered, not reckoned, not put to account in Demonstration. And I exemplified the same by the Body of the Earth describing the Ecciprique Line; because the magnitude is not there reckoned nor chargeth the Ecciprique Line with any breadth. But I perceive you understand not what the word *consideration* signifieth, but take it for comparison or relation; and say I ought to define a Point simply, and not by relation to a greater Body; as if to reckon, and to compare were the same thing. *Omnia mihi* (saith *Cicero*) *provisa & considerata sunt*. I have provided and reckoned every thing. There is a great difference between Reckoning and Relation.

Again, you ask why *Corpus motum*, a Body moved. He tell you; because the motion was necessary.

necessary for the generation of a Line. And though after the generation of the Line, the Point should rest, yet it is not necessary from this Definition, that it should be no more a Point; nor when Euclide defines a Sphere by the circumduction of a Semicircle upon an Axis that resteth, doth it follow from thence, when the Sphere, Axis, Center and all (as that of the Earth) is moved from place to place, that it is no more an Axis.

Lastly, you object that motion is accidentary to a Point, and consequently not essentiall, nor to be put into the Definition. And is not the circumduction of a Semicircle accidentary to a Sphere? Or do you think the Sphere of the Sun was generated by the revolution of a Semicircle? And yet it was thought no fault in Euclide to put that motion into the Definition of a Sphere.

The conceit, you have concerning Definitions, that they must explicate the essence of the thing defined, and must consist of a *genus*, and a *difference*, is not so universally true as you are made believe, or else there be very many insufficient Definitions that pass for good with you in Euclide. You are much deceived if you think these wofull notions of yours, and the Language that doth every where accompany them, shew handsomely together. Or that such grounds as these be able to sustain so many, and so haughty reproaches as you advance upon them, so as they fall not (as you shall see immediately) upon your own head. I say a point hath Quantity, but not to be reckoned in Demonstrating the properties of Lines, Solids, or Superficies; You say it hath no Quantity at all, but is plainly Nothing.

The first of the Petitions of Euclide is, that a Line may be drawn from Point to Point at any distance. The second, that a straight Line may be produced. The third, that on any Center a circle may be described at any distance. And the eighth Axiome (which Sir H. Savile observes to be the foundation of all Geometry) is this, *Qua sibi mutuo congruunt, &c. Those things that are applyed one to another in all Points, are equall.* All or any of these Principles being taken away, there is not in Euclide one Proposition Demonstrated, or Demonstrable. If a Point have not Quantity, a Line can have no Latitude; and because a Line is not drawn but by motion, by motion of a Body, and Body imprinteth Latitude all the way, it is impossible to draw or produce a straight Line, or to describe a circular Line without Latitude. Also, if a Line have no Latitude, one straight Line cannot be applyed to another. To them therefore that deny a Point to have Quantity, that is, a Line to have Latitude, the forenamed Principles are not possible, and consequently no proposition in Geometry is demonstrated, or demonstrable. You therefore that deny a Point to have Quantity, and a Line to have Breadth, have nothing at all of the Science of Geometry. The practise you may have; but so hath any man that hath learned the bare Propositions by heart; but they are not fit to be Professors either of Geometry, or of any other Science that dependeth on it. Some men perhaps may say that this controversy is not much worth, and that we both mean the same thing. But that man (though in other things prudent enough) knoweth little of Science and Demonstration. For Definitions are not onely used to give us the Notions of those things whose appellations are defined; for many times they that have no Science have the Ideas of things more perfect then such as are raised by Definitions. As who is there that understandeth not better what a straight Line is, or what Proportion is, and what many other things are without Definition, then some that set down the Definitions of them. But their use is, when they are truly and clearly made, to draw Arguments from them for the Conclusions to be proved. And therefore you that in your following censures of my Geometry, take your Argument so often from this, That a Point is nothing, and so often revile me for the contrary, are not to be allowed such an excuse. He that is here mistaken, is not to be called Negligent in his Expression, but Ignorant of the Science.

In the next place, you take exceptions to my Definition of *Equall Bodies*, which is this, *Corpora aequalia sunt quae eundem locum possidere possunt.* Equall bodies are those which may have the same place. To which you object impertinently, that I may as well define a man to be He that may be Prince of Transilvania, Wittily, as you count wit. Formerly in every Definition, you exacted an Explication of the Essence. You are therefore of opinion that the Possibility of being

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ing Prince of Transylvania, is no less Essentiall to a man, then the Possibility of the being of two Bodies successively in the same place, is Essentiall to Bodies equall.

You take no notice of the twenty third Article of this same Chapter, where I define what it is we call Essence; namely, that Accident, for which we give the thing its name. As the Essence of a man is his Capacity of reasoning, the Essence of a white-body, whiteness, &c. because we give the name of *man* to such Bodies as are capable of Reasoning, for that their capacity; and the name of *White* to such Bodies as have that colour, for that colour. Let us now examine why it is that men say Bodies are one to another Equall; and thereby we shall be able to determine whether the possibility of having the same place, be Essentiall or not to Bodies equall, and consequently whether this Definition be so like to the Defining of a man by the Possibility of being Prince of Transylvania, as you say it is. There is no man (besides such Egregious Geometricians as your selves) that inquireth the equality of two bodies, but by measure. And for Liquid Bodies, or the Aggregates of innumerable small Bodies, men (men I say) measure them by putting them one after another into the same vessel, that is to say, into the same place (as Aristotle defines place) or into the space determined by the vessel, as I define place. And the Bodies that so fill the vessel, they acknowledge, and receive for equall. But though, when hard Bodies cannot be so measured, without the incommodity, or trouble of altering their Figure, they then enquire (if the Bodies are both of the same-kind) their equality by weight, knowing (without your teaching) that equall bodies of the same nature, weigh Proportionably to their magnitudes; yet they do it not for fear of missing of the equality, but to avoid inconvenience, or trouble. But you, (you I say) that have no Definition of Equall, neither received from others, nor framed by your selves, out of your shallow meditation, and deep conceit of your own Wits, contend against the common light of Nature. So much is unheedy learning a hinderance to the knowledge of the truth, and changeth into Elves those that were beginning to be men.

Again, when men inquire the equality of two Bodies in length, they measure them by a common measure; in which measure they consider neither breadth nor thickness, but how the length of it agreeth; first, with the length of one of the Bodies, then with the length of the other. And both the Bodies whose lengths are measured, are successively in the same place under their common measure. Place therefore in Lines also, is the proper Index and discoverer of Equality, and Inequality. And as in length, so it is in breadth and thickness, which are but Lengths otherwise taken in the same Solid Body. But now when we come from this Equality, and Inequality of Lengths known by measure, to determine the Proportions of Superficies, and of Solids, by ratiocination, then it is that we enter into Geometry; for the making of Definitions, in whatsoever Science they are to be used, is that which we call *Philosophia prima*. It is not the work of a Geometrician, as a Geometrician, to Define what is Equality, or Proportion, or any other word he useth, though it be the work of the same man, as a man. His Geometricall part is, To draw from them, as many true and usefull Theoremes as he can.

You object secondly, That a Pyramis may be equall to a Cube, whilst it is a Pyramis. True, And so also whilst it is a Pyramis it hath a possibility by flexion and transposition of parts, to become a Cube, and to be put into the place where another Cube equall to it was before. This is to argue like a child that hath not yet the perfect understanding of any Language.

In the third and fourth objection, you teach me to define Equall Bodies (it I will needs define them by place) by the Equality of place, and to say that Bodies are Equall that have Equall places. Teach others, if you can, to measure their grain, not by the same, but Equall Bushels.

In the fifth objection, you except against the word *can*, in that I say that Bodies are Equall, which *can* fill the same place. For the greater Body *can* (you say) fill the place of the less, though not reciprocally the less of the greater; It is true, that though the place of the less, *can* never be the place of the greater yet it may be filled by a part of the greater. But 'tis not then the greater Body that filleth the place of the less, but a part of it, that is to say, a less Body. Howsoever.

to take away from simple men this straw they stumble at, I have now put the Definition of Equal Bodies into these words, *Equall Bodies are those whereof every one can fill the place of every other.* And if my Definition displease you, propound your own, either of *Equall Bodies*, or of *Equalls* simply. But you have none. Take therefore this of mine.

The sixth is a very admirable exception. *What (say you) if the same Body can sometimes take up a greater, sometimes a lesser place, as by Rarefaction and Condensation?* I understand very well that Bodies may be sometimes thin and sometimes thick, as they chance to stand closer together, or further from one another. So in the Mathematick-Schools, when you read your Learned Lectures, you have a thick or thronging Audience of Disciples, which in a great Church, would be but a very thin company. I understand how thick and thin may be attributed to bodies in the Plurall, as to a company, but I understand not how any one of them is thicker in the School, then in the Church; or how any one of them taketh up a greater room in the School (when he can get in) then in the street. For I conceive the Dimensions of the Body, and of the Place, whether the place be filled with Gold or with Air, to be coincident and the same; and consequently both the Quantity of the Air, and the Quantity of the Gold to be severally equal to the Quantity of the place; and therefore also (by the first Axiome of *Euclide*) equal to one another; insomuch, as if the same Air should be by Condensation contained in a part of the place it had, the dimensions of it would be the same with the dimensions of part of the place, that is, should be less then they were, and by consequence the Quantity less. And then either the same body must be less also, or we must make a difference between greater Bodies, and Bodies of greater Quantity; which no man doth that hath not lost his wits by trusting them with absurd teachers. When you receive Salary, if the Steward give you for every shilling a piece of six pence, and then say, every shilling is condensed into the room of a six pence, I believe you would like this Doctrine of yours much the worse. You see how by your ignorance you confound the affairs of mankind, as far forth as they give credit to your opinions, though it be but little. For nature abhorres even empty words, such as are (in the meaning you assign them) *Rarefying* and *Condensing*. And you would be as well understood if you should say (coining words by your own power) that the same Body might take up sometimes a greater, sometimes a lesser place, by *Wallsification* and *Wardenstation*, as by *Rarefaction* and *Condensation*. You see how admirable this your objection is.

In the seventh objection you bewray another kind of Ignorance, which is the Ignorance of what are the proper works of the severall parts of Philosophy. *Though it were out of doubt (say you) that the same Body cannot have several Magnitudes, yet seeing it is master of Natural Philosophy, nor hath any thing to do with the present business, to what purpose is it to mention it in a Mathematicall Definition?* It seems by this, that all this while you think it is a piece of the Geometry of *Euclide*, no less to make the Definitions he useth, then to infer from them the Theorems he demonstrateth. Which is not true. For he that telleth you in what sense you are to take the Appellations of those things which he nameth in his discourse, teacheth you but his Language, that afterwards he may teach you his Art. But teaching of Language is not Mathematick, nor Logick, nor Physick, nor any other Science; and therefore to call a Definition (as you do) Mathematicall, or Physicall, is a mark of Ignorance (in a Professor) unexcusable. All Doctrine begins at the understanding of words, and proceeds by Reasoning till it conclude in Science. He that will learn Geometry must understand the Termes before he begin, which that he may do, the Master demonstrateth nothing, but useth his Naturall prudence onely, as all men do, when they endeavour to make their meaning clearly known. For words understood are but the seed, and no part of the harvest of Philosophy. And this seed was it, which *Aristotle* went about to sow in his twelve Books of *Metaphysics*, and in his eight Books concerning the *Hearing of Naturall Philosophy*. And in these Books he defineth Time, Place, Substance or *Being*, Quantity, Relation, &c. that from thence might be taken the Definitions of the most generall words for Principles in the severall parts of Science. So that all Definitions proceed from common understanding; of which, if any man rightly write, he may properly call his

writing

writing *Philosophia prima*, that is, the Seeds, or the Grounds of Philosophy. And this is the Method I have used, defining Place, Magnitude, and the other the most generall Appellations in that part which I intitle *Philosophia prima*. But you now not understanding this, talk of Mathematicall Definitions. You will say perhaps that others do the same as well as you. It may be so. But the appeaching of others does not make your ignorance the less.

In the eighth place you object not, but ask me *why I define equall Bodies apart*. I will tell you. Because all other things which are said to be equall, are said to be so, from the equality of Bodies; as two lines are said to be equall, when they be coincident with the Length of one and the same Body; and equall Times, which are measured by equall Lengths of Body, by the same Motion. And the reason is, because there is no Subject of Quantity, or of Equality, or of any other accident but Body; all which I thought certainly was evident enough to any uncorrupted Judgement; and therefore, that I needed first to define Equality in the Subject thereof, which is Body, and then to declare in what sense it was attributed to Time, Motion, and other things that are not Body.

The ninth objection is an egregious cavill. Having set down the Definition of *Equall Bodies*, I considered that some men might not allow the attribute of Equality to any things, but those which are the Subjects of Quantity, because there is no Equality, but in respect of Quantity. And to speak rigidly, *Magnum & Magnitudo* are not the same thing; for that which is great, is properly a Body, whereof greatness is an Accident. In what sense therefore (might you object) can an Accident have Quantity? For their sakes therefore that have not Judgement enough to perceive in what sense men say the Length is so Long, or the Superficies so broad, &c. I added these words, *Eadem ratione (quâ scilicet corpora dicuntur equalia) Magnitudo Magnitudini equalis dicitur*, that is, in the same manner, as Bodies are said to be equal, their magnitudes also are said to be equall. Which is no more then to say, when Bodies are Equall, their Magnitudes also are called Equall. When Bodies are Equall in Length, their Lengths are also called Equall. And when Bodies are Equall in Superficies, their Superficies are also called Equall. All which is common speech, as well amongst Mathematicians, as amongst common people; and (though improper) cannot be altered, nor needeth to be altered to intelligent men. Nevertheless I did think fit to put in that clause, that men might know what it is we call Equality, as well in Magnitudes as in *Magnis*, that is, in Bodies. Which you so interpret, as if it bore this sense, that when Bodies are Equall, their Superficies also must be Equall, contrary to your own knowledge, onely to take hold of a new occasion of reviling. How unhandsome, and unmanly this is, I leave to be judged by any Reader that hath had the fortune to see the world, and converse with honest men.

Against the fourteenth Article, where I prove that the same Body hath alwayes the same magnitude, you object nothing but this, that though it be granted, that the same Body hath the same magnitude, while it resteth, yet I bring nothing to prove that when it changeth place, it may not also change its Magnitude by being enlarged or contracted. There is no doubt, but to a Body (whether at rest or in motion) more Body may be added, or part of it taken away. But then it is not the same Body, unless the Whole, and the Part be all one. If the Schools had not let your wit away, you could never have been so stupid as not to see the weakness of such objections. That which you add in the end of your objections to this eighth Chapter, that I allow not Euclide this Axiom gratis, that the Whole is greater then a Part, you know to be untrue.

At my eleventh Chapter, you enter into dispute with me, about the nature of Proportion. Upon the truth of your Doctrine therein, and partly upon the truth of your opinions concerning the Definitions of a Point, and of a Line, dependeth the Question whether you have any Geometry, or none; and the truth of all the Demonstrations you have in your other Books, namely, of the *Angle of Contact*, and *Arithmetica Infinitorum*. Here I say you enter, how you will get out (your reputation saved) we shall see hereafter.

When a man asketh what Proportion one Quantity hath to another, he asketh how great or how

how little the one is comparatively to, or in respect of the other. When a Geometrician prefixeth before his Demonstrations a Definition, he doth it not as a part of his Geometry, but of naturall evidence, not to be demonstrated by Argument, but to be understood in understanding the Language wherein it is set down; though the matter may nevertheless (if besides Geometry he have wit) be of some help to his Disciple to make him understand it the sooner. But when there is no significant Definition prefixed (as in this case, where Euclides Definition of Proportion, That it is a *whatsoever habitude of two Quantities*, &c. is insignificant, and you alledge no other) every one that will learn Geometry, must gather the Definition from observing how the word to be defined is most constantly used in common speech. But in common speech if a man shall ask how much (for example) is six in respect of four, and one man answer that it is greater by two, and another that it is greater by half of four, or by a third of six, he that asked the question, will be satisfied by one of them, though perhaps by one of them now, and by the other another time, as being the only man that knoweth why he himself did ask the Question. But if a man should answer, as you would do, that the Proportion of six to two is of those numbers, a certain Quotient, he would receive but little satisfaction. Between the said answers to this Question, How much is six in respect of four? there is this difference. He that answereth that it is more by two, compareth not two with four, nor with six, for two is the name of a Quantity absolute. But he that answereth it is more by half of four, or by a third of six, compareth the difference with one of the differing Quantities. For halves, and thirds, &c. are names of Quantity compar'd.

From hence there ariseth two Species or kinds of (*Ratio*) Proportion, into which the generall word *Proportion* may be divided. The one whereof (namely, that wherein the Difference is not compared with either of the differing Quantities) is called (*Ratio Arithmetica*) Arithmetically Proportion; the other (*Ratio Geometrica*) Geometrically Proportion, and (because this latter is only taken notice of by the name of Proportion) simply *Proportion*. Having considered this, I defined Proportion, Chap. 11. Arti. 3. in this manner, *Ratio est Relatio Antecedentis ad Consequens secundum magnitudinem*. Proportion is the Relation of the Antecedent to the Consequent in Magnitude; having immediately before defined Relatives, Antecedent, and Consequent in the same Article, and by way of explication added, that such Relation was nothing else but that one of the Quantities was equall to the other, or exceeded it by some Quantity, or was by some Quantity exceeded by it. And for exemplification of the same, I added further, that the Proportion of three to two, was that three exceeded two by a unity; but said not that the unity, or the difference whatsoever it were, was their proportion, for Unity, and to exceed another by Unity, is not the same thing. This is cleer enough to others. Let us therefore see why it is not so to you. You say I make Proportion to consist in that which remaineth after the lesser Quantity is subtracted out of the greater; and that you make it to consist in the Quotient, when one number is divided by the other. Wherein you are mistaken; first, in that you say I make the Proportion to consist in the Remainder. For I make it to consist in the act of exceeding, or of being exceeded, or of being equall; whereas the Remainder is alwayes an absolute Quantity, and never a Proportion. To be more or less then another number by two, is not the number two; Likewise to be equall to two, where the difference is nothing, is not that nothing? Again, you mistake in saying the Proportion consisteth in the Quotient. For divide twenty by five, the Quotient is four. Is it not absurd to say that the Proportion of five to twenty, or of twenty to five, is four? You may say the Proportion of five to twenty, is the Proportion of one to four. And so say I. And you may therefore also say, that the Proportion of one to four is a measure of the Proportion of five to twenty, as being Equall. And so say I. But that is only in Geometrically Proportion, and not in Proportion universally. For though the Species obtain the Denomination of the Genus; yet it is not the Genus. But as the Quotient giveth us a measure of the Proportion of the Dividend to the Divisor in Geometrically Proportion, so also the Remainder after Subtraction is the measure of Proportion Arithmetically.

You

You object in the next place, *That if the Proportion of one Quantity to another be nothing but the excess or defect, then, wheresoever the Excess or Defect is the same, there the Proportion is the same.* This you say follows in your Logick, and from thence, that the Proportion of three to two, and five to four is the same. But is not three to two, and five to four, where the Excess is the same number, the same Proportion Arithmetically? And is not Arithmetical Proportion, Proportion? You take here (*Ratio*) Proportion (which is the *Genus*) for that *Species* of it, which is called Geometrical, because usually this *Species* has the name of Proportion simply. Also that the Proportion of three to two, is the same with that of nine to six; Is it not because the excesses are one and three, the same portions of three and nine, that is to say, the same excesses comparatively? I wonder you ask me not what is the *Genus* of Arithmetically, and Geometrical Proportions; and what the *Difference*. The *Genus* is (*Ratio*) Proportion, or Comparison in Magnitude, and the *Difference* is that one Comparison is made by the absolute Quantity, the other by the Comparative Quantity, of the Excess or Defect, if there be any. Can any thing be clearer than this? You after come in with *Ignosce hebitudini* to no purpose. I am not so inhumane as not to pardon dulness, or madness; They are not voluntary faults. But when men adventure voluntarily, to talk of that they understand not censoriously and scornfully, I may tell them of it.

This difference between the Excesses or Defects, as they are simply or comparatively reckoned, being thus explained, all the rest of that you say in your objections to this eleventh Chapter (saying that *Art. 5. for Ratio binarii ad quinarium est superari Ternario*, as it is in other places, I have put too hastily *Ratio binarii ad quinarium est Ternarium*) will be understood by every Reader to be frivolous, and to proceed from the ignorance of what Proportion is.

At the twelfth Chapter you only note that I say, *That the Proportion of Inequality is Quantity, but the Proportion of Equality not Quantity*, and refer that which you have to say against it to the Chapter following; to which place, I shall also come in the following Lesson.

Of the Faults that Occurre in Demonstration.

To the same egregious Professors of the Mathematics in the University of Oxford.

LESSON III.

YOU begin your reprehension of my thirteenth Chapter with a Question. Whereas I divide Proportion into Arithmetically, and Geometrical; You ask me what proportion it is I so divide. Euclide divides an Angle into Right, Obtuse, and Acute. I may ask you as pertinently what Angle it is he so divides? Or when you divide *Animal* into *Homo*, and *Brutum*, what Animal that is which you so divide? You see by this how absurd your Question is. But you say the Definition of Proportion which I make at Chap. 11. Art. 3. namely, that Proportion

portion is the comparison of two Magnitudes, one to another, agreeth not, neither with Arithmetical, nor with Geometrical Proportion. I believe you thought so then, but having read what I have said in the end of the last Lesson, if you think so still, your fault will be too great, to be pardoned easily. But why did you think so before? Is it not because there was no Definition in *Euclide* of Proportion universall, and because he maketh no mention of Proportion Arithmetical, and because you had not in your minds a sufficient notion thereof your selves to supply that Defect? And is not this the cause also, why you put in this Parenthesis (if Arithmetical Proportion, ought to be called Proportion)? which is a confession that you know not whether there be such a thing as Arithmetical Proportion or not; notwithstanding, that on all occasions, you speak of Arithmetically Proportionals. Yes, this was it that made you think that Proportion universall, and Proportion Geometrical is the same and yet to say you cannot tell whether they be the same or not. 'Tis no wonder therefore, if in such confusion of the understanding, you apprehend not that the Proportions of two to five, and nine to twelve, are the same; for you are blinded by seeing that they are not the same Proportions Geometrical. Nor doth it help you that I say the Difference is the Proportion, for by Difference you might if you would, have understood the act of Differing.

At the second Article you note for a fault in Method, that after I had used the words *Antecedent and Consequent of a Proportion in some of the precedent Chapters, I define them afterwards*. I do not believe you say this against your knowledge, but that the eagerness of your malice made you oversee. Therefore go back again to the third Article of Chap. 11. Where having defined Correlatives, I add these words, *Of which the first is called the Antecedent, the second the Consequent*. This is but an oversight, though such as in me, you would not have excused.

At the thirteenth Article you find fault with, that I say that the Proportion of Inequality, whether it be of Excess, or of Defect, is Quantity, but the Proportion of Equality is not Quantity. Whether that which you say, or that which I say be the truth, is a Question worthy of a very strict Examination. The first time I heard it argued, was in *Mersennus* his Chamber at Paris, at such time, as the first volume of his *Cogitata Physico-Mathematica* was almost printed: In which, because he had not said all he would say of Proportion, he was forced to put the rest into a Generall Preface; which as was his custom, he did read to his Friends, before he sent it to the Press. In that generall Preface under the Title *de Rationibus atq; Proportionibus*, at the Numbers twelve, thirteen, fourteen, he maintaineth against *Clavius*, that the Composition of Proportions is (as of all other things) a Composition of the Parts to make a Totall; and that the proportion of equality answereth in Quantity, to non-ens, or Nothing; the proportion of excess, to ens, or Quantity; and the Proportion of Defect, to less then Nothing; because Equality (he saies) is a term of middle signification, between Excess and Defect. And there also he refuteth the Arguments which *Clavius* at the end of the ninth Element of *Euclide* bringeth to the contrary. And though this were approved by divers good Geometricians then present, and never gain-sayed by any since, Yet do not I say it upon the credit of them, but upon sufficient grounds. For it hath been demonstrated by *Eutocius* that if there be three magnitudes, the proportion of the first to the third is compounded of the proportions of the first to the second, and of the second to the third; Which also I demonstrate in this Article. And if there were never so many magnitudes ranked, it might be likewise demonstrated, that the Proportion of the first to the last is compounded of the Proportions of the first to the second, and of the second to the third, and of the third to the fourth, and so on to the last. If therefore we put in order any three numbers, whereof the two last be equal, as four, seven, seven, the Proportion of four the first, to seven the last, will be compounded of the Proportions of four the first, to seven the second, and of seven the second, to seven the third. Wherefore the Proportion of seven to seven (which is of equality) addeth nothing to the Proportion of four the first, to seven the second; and consequently the Proportion of seven to seven hath no Quantity. But that the Proportion of Inequality hath Quantity, I prove it from this, that one Inequality may be greater then another.

But

But for the clearing of this Doctrine (which *Mersennus* calls Intricate) of the composition of Proportions, I observed, first, that any two Quantities, being exposed to sense, their Proportion was also exposed; which is not Intricate. Again, I observed that if besides the two exposed Quantities, there were exposed a third, so as the first were the least, and the third the greatest, or the first the greatest, and the third the least, that not only the Proportions of the first to the second, but also (because the Differences, and the Quantities proceed the same way) the Proportion of the first to the last is exposed by composition, or addition of the Differences; nor is there any intricacy in this. But when the first is less than the second, and the second greater than the third, or the first greater than the second, and the second less than the third, so that to make the first and second equal, if we use addition, we must to make the second and third equal use subtraction, then comes in the intricacy, which cannot be extricated, but by such as know the truth of this Doctrine which I now delivered out of *Mersennus*: namely, That the Proportions of Excess, Equality, and Defect, are as Quantity, not-Quantity, nothing want Quantity, or as Symbolists mark them $0 + 1. 0. 0 - 1$ And upon this ground I thought depended the universal truth of this Proposition, that in any rank of Magnitudes of the same kind, the Proportion of the first to the last, was compounded of all the Proportions (in order) of the intermediate Quantities; the want of the proof thereof Sir *Henry Savile* calls (*Nævus*) a mole in the Body of Geometry. This Proposition is demonstrated at the thirteenth Article of this Chapter.

But before we come thither, I must examine the Arguments you bring to confute this Proposition, that the Proportion of Inequality is Quantity, of Equality not Quantity.

And first, you object that Equality and Inequality are in the same Predicament. A pretty Argument to flesh a young Scholar in the Logick School, that but now begins to learn the Predicaments. But what do you mean by *Æquale*, and *Inæquale*? Do you mean *Corpus Æquale*, and *Corpus Inæquale*? They are both in the Predicament of Substance, neither of them in that of Quantity; Or do you mean *Æqualitas*, and *Inæqualitas*? They are both in the Predicament of Relation, neither of them in that of Quantity, and yet both *Corpus*, and *Inæqualitas*, though neither of them be Quantity, may be *Quanta*, that is, both of them have Quantity. And when men say Body is Quantity, or Inequality is Quantity, they are no otherwise understood, then if they had said *Corpus est tantum*, and *Inæqualitas tanta*, not *Tantitas*; that is, Bodies and Inequalities are so much, not somuchness; and all intelligent men are contented with that expression, and your selves use it. And the Quantity of Inequality is in the Predicament of Quantity, because the measure of it is in that Line by which one Quantity exceeds the other. But when neither exceedeth other, then there is no Line of Excess, or Defect by which the Equality can be measured, or said to be so much, or be called Quantity. Philosophy teacheth us how to range our words; but *Aristotles* ranging them in his Predicaments, doth not teach Philosophy; And therefore no Argument taken from thence, can become a Doctor, and a Professor of Geometry.

To prove that the Proportion of Inequality was Quantity, but the Proportion of Equality, not Quantity, my Argument was this; That because one Inequality may be greater or less than another, but one Equality cannot be greater nor less than another, Therefore Inequality hath Quantity, or is *Tanta*, and Equality not. Here you come in again with your Predicaments, and object, that to be susceptible of *magis* and *minus* belongs not to quantity, but to Quality; but without any proof, as if you took it for an Axiome. But whether true or false, you understand not in what sense it is true or false. 'Tis true, that one Inequality is Inequality, as well as another; as one heat is heat as well as another; but not as great; *Tam*, but not *Tantus*. But so it is also in the Predicament of Quantity; one Line is as well a Line as another, but not so great. All degrees, intensions, and remissions of Quality, are greater or less Quantity of force, and measured by Lines, Superficies, or Solid Quantity, which are properly in the Predicament of Quantity. You see how wise a thing it is to argue from the Predicaments of *Aristotle*, which you understand not. And yet you pretend to be less added to the authority of *Aristotle*, now, then heretofore.

In the next place you say, I may as well conclude from the not susception of *greater* and *less*, that a Right Angle is not Quantity, but an Oblique one is. Very learnedly. As if to be *greater* or *less* could be attributed to a Quantity once determined. Number (that is number indefinitely taken) is susceptible of *greater* and *less*, because one number may be greater then another. And this is a good Argument to prove that Number is Quantity. And do you think the Argument the worse for this, that one six cannot be greater then another six? After all these childish Arguments which you have hitherto urged, can you persuade any man, or your selves, that you are Logicians?

To the fifth and sixth Article you object, first, that if I had before sufficiently Defined (*Ratio*). *Proportion*, I needed not again define what is (*eadem Ratio*) the same *Proportion*; and ask me whether when I have defined man, I use to define anew what is the same man? You think when you have the Definition of *Homo*, you have also the Definition of *idem Homo*, when 'tis harder to conceive what *idem* signifies, then what *Homo*. Besides, *idem* hath not the same signification alwayes, and with whatsoever it be joyned; it doth not signifie the same with *Homo*, that it doth with *Ratio*. For with *Homo*, it signifies the same individual man, but with *Ratio* it signifies a like, or an equall *Proportion*. And both (*Ratio*) *Proportion*, and (*idem*) the same, being defined, there will still be need of another Definition for (*eadem Ratio*) the same *Proportion*. And this is enough to defend both my self and *Euclide*, against this objection. For *Euclide* also after he had Defined (*Ratio*) *Proportion*, and that succinctly as he believed, yet he defines the same *Proportion* again apart. I know you did not mean in this place to object any thing against *Euclide*; but you saw not what you were doing. There is within you some speciall cause of Intenebation, which you should do well to look to.

In the next place you say, when I had defined Arithmetical Proportions to be the same when the difference is the same; it was to be expected I should define Geometrical Proportions to be then the same, when the Antecedents are of their Consequents *Totuple*, or *Tantuple*, that is, equimultiple (for *Tantuplum* signifies nothing.) In plain words, you expected, that as I defined one by subtraction, I should define the other by the Quotient in Division. But why should you expect a Definition of the same *Proportion* by the Quotient? Neither Reason nor the Authority of *Euclide* could move you to expect it. Or why should you say it was to be expected? But it seems you have the vanity to place the measure of truth in your own Learning. In Lines incommensurable there may be the same *Proportion*, when nevertheless there is no Quotient; for setting their Symboles one above another doth not make a Quotient; for Quotient there is none, but in aliquot parts. It is therefore impossible to define *Proportion* universally, by comparing Quotients. This incommensurability of Magnitudes was it that confounded *Euclide* in the framing of his Definition of *Proportion* at the fifth Element. For when he came to numbers, he defined the same *Proportion* ineprehensibly thus, Numbers are then *Proportionall*, when the first of the second, and the third of the fourth are equimultiple, or the same part, or the same parts; and yet there is in this Definition no mention at all of a Quotient. For though it be true that if in dividing two Numbers you make the same Quotient, the Dividends and the Divisors are *Proportionall*, yet that is not the Definition of the same *Proportion*, but a Theoreme Demonstrable from it. But this Definition *Euclide* could not accommodate to *Proportion* in General, because of incommensurability.

To supply this want, I thought it necessary to seek out some way, whereby the *Proportion* of two Lines, Commensurable, or Incommensurable, might be continued perpetually the same. And this I found might be done by the *Proportion* of two Lines described by some uniform motion, as by an Efficient cause both of the said Lines, and also of their *Proportions*. Which motions continuing, the *Proportions* must needs be all the way the same. And therefore I defined those Magnitudes to have the same Geometrical *Proportion*, when some cause producing in equal times, Equall Effects, did determine both the *Proportions*. This you say needs an *Oedipus* to make it understood. You are (I see) no *Oedipus*; but I do not see any difficulty, neither in the Definition, nor in the Demonstration. That which you call perplexity in the

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Explication, is your prejudice, arising from the Symboles in your fancy. For men that pretend no less to naturall Philosophy, then to Geometry, to find fault with bringing Motion and Time into a Definition, when there is no effect in nature, which is not produced in Time by Motion, is a shame. But you swim upon other mens bladders in the Superficies of Geometry, without being able to indure diving. Which is no fault of mine; and therefore I shall (without your leave) be bold to say, I am the first that hath made the grounds of Geometry firm and coherent. Whether I have added any thing to the Edifice or not, I leave to be judged by the Readers. You see, you that profess with the pricking of bladders the letting out of their vapour, how much you are deceived. You make them swell more then ever.

For the Corollaries that follow this sixth Article, you say they contain nothing new. Which is not true. For the ninth is new, and the Demonstrations of all the rest are new, being grounded upon a new Definition of Proportion, and the Corollaries themselves for want of a good Definition of Proportion, were never before exactly Demonstrated. For the truth of the sixth Definition of the fifth Element of *Euclide* cannot be known but by this Definition of mine, because it requires a Triall in all numbers possible; that is to say, an infinite time of Triall, whether the quimultiples of the first and third, and of the second and fourth in all multiplications do together exceed, together come short, and are together equall; which Triall is impossible.

In objecting against the thirteenth and sixteenth Article, I observe that you bewray together, both the greatest Ignorance, and the greatest malice; and 'tis well, for they are suitable to one another, and fit for one and the same man. In the thirteenth Article my Proposition is this, *If there be three Magnitudes that have Proportion one to another, the Proportions of the first to the second, and of the second to the third taken together (as one Proposition) are equall to the Proportion of the first to the third.* This Demonstrated, there is taken away one of those Moles which Sir Henry Savile complaineth of in the Body of Geometry. Let us see now what you say both against the Enunciation, and against the Demonstration.

Against the Enunciation you object, that other men would say (not the Proportions of the first to the second, and of the second to the third, taken together, &c. but) *the Proportion which is compounded of the Proportion of the first to the second, and of the second to the third, &c.* Is not the compounding of any two things whatsoever the finding of the sum of them both, or the taking of them together as one totall. This is that absurdity of which *Mersennus* in the generall Preface to his *Cogitata Physico-Mathematica* hath convinced *Clavius*, who at the end of *Euclides* ninth Element denyeth the composition of Proportion to be a Composition of Parts to make a Totall; Which therefore he denied, because he did not observe, that the Addition of a Proportion of defect, to a Proportion of Excess, was a Substraction of Magnitude; and because he understood not that to say, Composition is not the making a whole of Parts, was contradictions; which all, but too learned men would as soon as they heard, abhorre. Therefore in saying that other men would not speak in that manner, you say in effect they would speak absurdly. You do well to mark what other Geometricians say; but you would do better if you could by your own Meditation, upon the things themselves, examine the truth of what they say. But you have no minde (you say) to contend about the Phrase. Let us therefore see what it is you contend about.

The Proportion (you say) which is compounded of double and triple Proportion, is not (as I would have it) Quintuple, but Sextuple, as in these numbers, six, three, one; where the Proportion of six to three is double, the Proportion of three to one triple, and the Proportion of six to one sextuple, not quintuple. Tell me (egregious Professors) how is six to three double Proportion? Is six to three the double of a number, or the double of some Proportion? All men know the number six is double to the number three, and the number three triple to an unity. But is the Question here of compounding numbers, or of compounding Proportions: *Euclide* at the last Proposition of his ninth Element says indeed, that these numbers, one, two, four, eight, are *ἑν διπλασίονι ἀναλογίᾳ* in double Proportion, yet there is no man that understands it otherwise, then if he had said in Proportion of the single Quantity, to the double Quantity,

ity; and after the same Rate, if he had said three, nine, twenty-seven, &c. had been in triple Proportion, all men would have understood it, of the Proportion of any Quantity to its Triple. Your instance therefore of six, three, one, is here impertinent, there being in them no doubling, no tripling, nor sextupling of Proportions, but of numbers. You may observe also that Euclid never distinguisheth between double and duplicate, as you do. One word διπλασιον serves him every where for either. Though I confess some curious Grammarians take διπλασιον for duplicate in number, and διπλοῦν for double in Quantity; which will not serve your turn. Your Geometry is not your own, but you ease your selves with Euclides; in which, as I have shewed you, there be some few great holes; and you by misunderstanding him, as in this place, have made them greater. Though the beasts that think your railing, roaring, have for a time admired you; yet now that through these holes of your case, I have shewed them your ears, they will be less affrighted. But to exemplify the composition of Proportions, take these numbers, thirty-two, eight, one, and then you shall see that the Proportion of thirty-two to one is the sum of the Proportions of thirty-two to eight, and of eight to one. For the Proportion of thirty-two to eight is double the Proportion of thirty-two to sixteen. And the Proportion of eight to one, is triple the Proportion of thirty-two to sixteen, and the Proportion of thirty-two to one is Quintuple of thirty-two to sixteen. But double and triple added together maketh quintuple. What can be here denied?

My Demonstration consisteth of three cases. The first is when both the Proportions are of defect, which is then, when the first Quantity is the least; as in these three Quantities, A B, A C, A D. The first case I demonstrated thus.

$$\begin{array}{c} A \quad B \quad C \quad D \\ a \end{array}$$

Let it be supposed that the point A were moved uniformly through the whole line A D. The Proportions therefore of A B to A C, and of A C to A D, are determined by the difference of the Times in which they are described. And the Proportion also of A B to A D, is that which is determined by the difference of the Times, in which they are described; but the difference of the Times in which A B and A C are described, together with the difference of the Times, wherein A C and A D are described, is the same with the difference of the Times, wherein are described A B and A D. The same cause therefore which determines both the Proportions of A B to A C, and of A C to A D, determines also the Proportion of A B to A D. Wherefore by the Definition of the same Proportion, Article six, the Proportion of A B to A C, together with the Proportion of A C to A D, is the same with the Proportion of A B to A D.

Consider now your argumentation against it. Let there be taken (say you) between A and B the Point c, and then in your own words, I argue thus. The difference of the Times wherein are described A B and A C, together with the difference of the Times, wherein are described A C and A D, is the same with the difference of the Times, in which are described A B and A D (namely B D, or B C + C D) wherefore the same cause which determines the two Proportions of A B to A C, and of A C to A D, determines also the Proportion of A B to A D. Let me ask you here whether you suppose the Motion from a to B, or from a to D, to have the same swiftness with the motion from A to B, or from A to D. If you do not, then you deny the supposition. If you do, then B C which is the difference of the Times A B and A C, cannot be the difference of the Times in which are described A B and A C, except A B and A C are equal. Let any man judge now whether there be any Paralogism in Orontius that can equal this. And whether all that follows in the rest of this, and the next two whole Pages, be not all a kind of raving upon the ignorance of what is the meaning of Proportion, which you also make more ill-favoured by writing it; not in language, but in Gamboles, I mean in the Symbols, which have made you call those demonstrations short, which put into words so many as a true demonstration requires, would be longer than any of those of Clavius upon the twelfth Element of Euclid.

To the sixteenth Article you bring no Argument, but fall into a loud Onocethmus (the special Figure wherewith you grace you Oratory) offended with my unexpected crossing of the Doctrine

Since you teach, that Proportion consisteth in a Quotient: For that being denyed your your
 $\frac{a}{b} = \frac{c}{d} + \frac{e}{f} = \frac{g}{b} + \frac{i}{k}$ comes to nothing, that is, to just as much as it is worth

But are not you very simple men, to say that all Mathematicians speak so, when it is not speaking? When did you see any man but your selves publish his Demonstrations by signs not generally received, except it were not with intention to demonstrate, but to teach the use of Signs? Had Pappus no Analytiques? Or wanted he the wit to shorten his reckoning by Signs? Or has he not proceeded Analytically in an hundred Problems (especially in his seventh Book) and never used Symboles? Symboles are poor unhandsome (though necessary) scaffolds of Demonstration; and ought no more to appear in publique, then the most determind necessary business which you do in your Chambers. But why (say you) is this limitation to the Proportion of the greater to the less? He tell you; because iterating of the Proportion of the less to the greater, is a making of the Proportion less, and the defect greater. And it is absurd to say that the taking of the same Quantity twice should make it less. And thence it is, that in Quantities, which begin with the less, as one, two, four, the Proportion of one to two, is greater then that of one to four; as is Demonstrated by Euclide Elem. 5. Prop. 8. and by consequent the Proportion of one to four, is a Proportion of greater littleness then that of one to two. And who is there, that when he knoweth that the respective greatness of four to one, is double to that of the respective greatness of four to two, or of two to one, will not presently acknowledge that the respective greatness of one to two, or two to four is double to the respective greatness of one to four. But this was too deep for such men as take their opinions not from weighing, but from reading.

Lastly, you object against the Corollarie of Art. 28. (which you make absurd enough by rehearsing it thus) *si quantitas aliqua divisa supponatur in partes aliquot aequales numero infinitas*. &c. Do you think that of *partes aliquot*, or of *partes aliquotas*, it can be said without absurdity, that they are *numero infinitas*? And then you say I seem to mean, that if of the Quantity A B, there be supposed a part C B, infinitely little, and that between A C and A B be taken two means, one Arithmetical A E, the other Geometrical A D, the difference between A D, and A E, will be infinitely little. My meaning is, and is sufficiently expressed, that the said means taken every where (not in one place onely) will be the same throughout. And you that say there needed not so much pains to prove it, and think you do it shorter, prove it not at all. For why may not I pretend against your demonstration, that B E the Arithmetical difference, is greater then B D the Geometrical difference. You bring nothing to prove it, and if you suppose it, you suppose the thing you are to prove. Hitherto you have proceeded in such manner with your *Elenchus*, as that so many objections as you have made, so many false Propositions you have advanced. Which is a peculiar excellence of yours, that for so great a stipend as you receive, you will give place to no man living for the number and grossness of errors you teach your Scholars.

At the fourteenth Chapter your first exception is to the second Article; where I define a plain in this manner. *A plain Superficies is that which is described by a straight Line moved, as that every Point thereof describe a severall straight Line*. In which you require, first, that instead of *describe*, I should have said *can describe*. Why do you not require of Euclide in the Definition of a Cone, instead of (*Continetur*) *is contained*, he say (*contineri potest*) *can be contained*? If I tell you how one Plain is generated, cannot you apply the same generation to any other Plain? But you object that the Plain of a Circle may be generated by the motion of the Radius, whose every point describeth not a straight but a crooked Line, wherein you are deceived; for you cannot draw a Circle (though you can draw the perimeter of a Circle) but in a Plain already generated. For the motion of a straight Line, whose one Point resting, describeth with the other Points severall perimeters of Circles, may as well describe a Conique Superficies, as a Plain. The Question therefore is, how you will in your Definition take in the Plain which must be generated before you begin to describe your Circle, and before you know what Point to make your Center. This objection therefore is to no purpose; and besides, that it

it reflecteth upon the perfect definitions of *Euclide* before the eleventh Element; it cannot make good his Definition (which is nothing worth) of a Plain Superficies, before his first Element.

In the next place you reprehend briefly this *Corollarie*, that two *Plaines* cannot inclose a *Solid*. I should indeed have added, *with the base on whose extrems they insist*. But this is not a fault to be ashamed of. For any man by his own understanding might have mended my expression without departing from my meaning. But from your Doctrine that a *Superficies* has no thickness, 'tis impossible to include a *Solid*, with any Number of *Plains* whatsoever; unless you say that *Solid* is included which nothing at all includes.

At the third Article, where I say of *crooked lines*, some are every where crooked, and some have parts not crooked. You ask me what crooked Line has parts not crooked; and I answer, it is that Line which with a straight Line makes a rectilineall Triangle. But this you say cannot stand with what I said before, namely, that a straight and crooked line cannot be coincident; which is true, nor is there any contradiction; for that part of a crooked line which is straight, may with a straight line be coincident.

To the fourth Article, where I define the Center of a Circle to be that Point of the Radius, which in the description of the Circle is unmoved; You object as a contradiction, that I had before defined a Point to be the body which is moved in the description of a Line. Foolishly, As I have already shown at your objection to Chap. 8. Art. 12.

But at the sixth Article, where I say that *crooked*, and *incongruous Lines* touch one another but in one Point, you make a cavill from this, that a Circle may touch a Parabola in two Points. Tell me truly, did you read and understand these words that followed, a crooked Line cannot be congruent with a straight line, because if it could, one and the same line should be both straight and crooked? If you did, you could not but understand the sense of my words to be this; when two crooked lines which are incongruous, or a crooked and a straight line touch one another, the contact is not in a Line, but only in one Point; and then your instance of a Circle and a Parabola, was a wilfull cavill, not befitting a Doctor. If you either read them not, or understood them not, it is your own fault. In the rest that followeth upon this Article, with your Diagram, there is nothing against me, nor any thing of use, novelty, subtilty or learning.

At the seventh Article, where I define both an *Angle*, simply so called, and an *Angle of Contingence*, by their severall generations, namely, that the former is generated when two straight Lines are coincident, and one of them is moved, and distracted from the other by circular motion upon one common Point resting, &c. You ask me to which of these kinds of Angle, I refer the Angle made by a straight Line when it cuts a crooked Line. I answer easily and truly, to that kind of Angle which is called simply an Angle. This you understand not. For how (will you say) can that Angle which is generated by the divergence of two straight Lines, be other then Rectilineall? O. how can that Angle which is not comprehended by two straight Lines, be other then Curvilineall? I see what it is that troubles you, namely, the same which made you say before, that if the Body which describes a Line be a Point, then there is nothing which is not moved that can be called a Point. So you say here, If an Angle be generated by the motion of a straight Line, then no Angle so generated can be Curvilineall. Which is as well argued, as if a man should say, the House was built by the carriage and motion of Stone and Timber, therefore when the carriage and that motion is ended, it is no more a house. Rectilineall and Curvilineall hath nothing to do with the nature of an Angle simply so called, though it be essentially to an Angle of Contact. The measure of an Angle simply so called is a circumference of a Circle, and the measure is alwayes the same kind of Quantity with the thing measured. The Rectitude or Curvity of the Lines which drawn from the Center intercept the Arch, is accidental to the Angle, which is the same, whether it be drawn by the motion circular of a straight line or of a crooked. The Diameter and the Circumference of a Circle make a right Angle, and the same which is made by the Diameter and the Tangent. And because the point of Contact is not (as you think) nothing, but a line unreckoned, and common both to the Tangent, and the Circum-

Circumference, the same Angle computed in the Tangent is Rectilineall, but computed in the Circumference, not Rectilineall, but mixt; or, if two Circles cut one another, Curvilineall. For every Chord maketh the same Angle with the Circumference which it maketh with the line that toucheth the Circumference at the end of the Chord. And therefore when I divide an Angle simply so called into Rectilineall, and Curvilineall, I respect no more the generation of it, then when I divide it into Right and Oblique. I then respect the generation, when I divide an Angle into an Angle simply so called, and an Angle of Contact. Thus that I have now said, if the Reader remember when he reads your objections to this, and to the ninth Article, he will need no more to make him see that you are utterly ignorant of the nature of an Angle, and that if ignorance be madness, not I, but you are mad; and when an Angle is comprehended between a straight and a crooked Line (if I may compute the same Angle as comprehended between the same straight Line and the Point of Contact) that it is conso- nant to my definition of a Point by a *Magnitude not considered*. But when you in your treatise *de Angulo Contactu* Chap. 3. Pag. 6. Lin. 8. have these words, *Though the whole concurrent Lines incline to one another, yet they form no Angle any where but in the very point of concurrence*, You, that deny a Point to be any thing, tell me how two nothings can form an Angle; or if the Angle be not formed neither before the concurrent Lines meet, nor in the Point of concurrence, how can you apprehend that any Angle can possibly be framed. But I wonder not at this absurdity, because this whole treatise of yours is but one absurdity continued from the beginning to the end; as shall then appear when I come to answer your objections to that which I have briefly and fully said of that Subject in my 14. Chapter.

At the twelfth Article I confess your exception to my universall definition of Parallels to be just, though insolently set down. For it is no fault of ignorance (though it also infect the demonstration next it) but of too much security. The Definition is this: *Parallels are those Lines or Superficies, upon which two straight Lines falling, and wheresoever they fall, making equal Angles with them both, are equal*; which is not, as it stands, universally true. But inserting these words *the same way*, and making it stand thus, *Parallel Lines or Superficies are those, upon which two straight Lines falling the same way, and wheresoever they fall, making equal Angles, are equal*, it is both true and universall; and the following Consec- tary with very little change, as you may see in the translation, perspicuously demonstrated. The same fault occureth once or twice more; and you triumph unreasonably, as if you had given therein a very great proof of your Geometry.

The same was observed also upon this place by one of the prime Geometricians of Paris, and noted in a Letter to his friend in these words, Chap. 14. Art. 12. *the Definition of Parallels wanteth somewhat to be supplied*. And of the Consecutory, he says, *it concludeth not, because it is grounded on the Definition of Parallels*. Truly, and severely enough, though without any such words as savour of Arrogance, or of Malice, or of the Clown.

At the thirteenth Article you recite the Demonstration by which I prove the Perimeters of two Circles to be Proportionall to their Semidiameters; and with *Esso, fortasse, recte, omnino*, nodding to the severall parts thereof, you come at Length to my last interence; Therefore by (Chap. 13. Art. 6.) the Perimeters and Semidiameters of Circles are Proportionall; which you deny; and therefore deny, because you say it followeth by the same Ratiocination, that Circles also and Spheres are Proportionall to their Semidiameters. For the same distance (you say) of the Perimeter from the Center which determines the magnitude of the Semidia- meter, determines also the magnitude both of the Circle, and of the Sphere. You acknow- ledge that Perimeters and Semidiameters have the cause of their determination such as in equal times make equal spaces. Suppose now a Sphere generated by the Semidiameters, whilst the Semicircle is turned about. There is but one Radius of an infinite number of Radii, which describes a great Circle, all the rest describe lesser Circles Parallel to it, in one and the same time of Revolution. Would you have men believe, that describing greater and lesser Circles, is ac- cording to the supposition (*temporibus aequalibus aequalia facere*) to make equal spaces in equal

times? Or when by the turning about of the Semidiameter is described the Plain of a Circle does it (think you) in equall times make the Plains of the Interior Circles equall to the plains of the exterior? Or is the *Radius* that describes the inner Circles equall to the *Radius* that describes the exterior? It does not therefore follow from any thing I have said in this demonstration, that either Spheres, or Plains of Circles, are Proportionall to their *Radii*. And consequently all that you have said, triumphing in your own Incapacity, is said imprudently by your selves to your own disgrace. They that have applauded you, have reason by this time to doubt of all the rest that follows, and if they can, to dissemble the opinion they had before of your Geometry. But they shall see before I have done, that not only your whole *Elenchus*, but also your other Books of the *Angle of Contact*, &c. are meer ignorance and gibberish.

To the fourteenth Article you object, that (in the sixth figure) I assume *gratis*, that FG , DE , BC , are Proportionall to AF , AD , AB ; and you referre it to be judged by the Reader. And to the Reader I referre it also. The not exact drawing of the Figure (which is now amended) is it that deceived you. For AF , FD , DB , are equall by construction. Also AG , GE , EC , are equall by construction. And FG , DK , BH , KE , HI , IC , are equall by Parallelism. And because AF , FG , are as the velocities wherewith they are described; also $2 AF$ (that is AD) and $2 FG$ (that is DE) are as the same velocities. And finally $3 AF$ (that is AB) and $3 FG$ (that is BC) are as the same velocities. It is not therefore assumed *gratis*, that FG , DE , BC , are Proportionall to AF , AD , AB , but grounded upon the sixth Article of the thirteenth Chapter; and consequently your objection is nothing worth. You might better have excepted to the placing of DE , first at adventure, and then making AD , two thirds of AB ; for that was a fault, though not great enough to trouble a Candid Reader; yet great enough, to be a ground, to a malicious Reader, of a Cavill.

That which you object to the third *Corollarie* of Art. 15, was certainly a dream. There is no assuming of an Angle CDE , for an Angle HDE , or BDE , neither in the Demonstration, nor in any of the *Corollaries*. It may be you dream't of somewhat in the twentieth Article of Chap. 16. But because that Article though once printed, was afterwards left out, as not serving to the use I had designed it for, I cannot guess what it is. For I have no Copy of that Article, neither printed nor written, but am very sure, though it were not usefull, it was true.

Article the sixteenth. Here we come to the Controversie concerning the *Angle of Contact*, which (you say) you have handled, in a speciall Treatise published; and that you have clearly demonstrated in your publick Lectures, that Peletarius was in the right. But that I agree not sufficiently neither with Peletarius, nor with Clavius. I confess I agree not in all points with Peletarius, nor in all points with Clavius. It does not thence follow that I agree not with the Truth. I am not (as you) of any faction, neither in Geometry, nor in Politicks. If I think that you, or Peletarius, or Clavius, or Euclide have erred, or been too obscure; I see no cause, for which I ought to dissemble it. And in this same Question, I am of opinion that Peletarius did not well in denying the *Angle of Contingence* to be an *Angle*. And that Clavius did not well to say the *Angle of a Semicircle* was less then a *Right-lined Right Angle*. And that Euclide did not well to leave it so obscure what he meant by *Inclination* in the Definition of a *Plain Angle*, seeing else where he attributeth *Inclination* onely to *Acute Angles*, and scarce any man ever acknowledged *Inclination* in a straight Line, to any other Line, to which it was perpendicular. But you in this Question of what is *Inclination*, though you pretend not to depart from Euclide, are nevertheless more obscure then he; and also are contrary to him. For Euclide by *Inclination* meaneth the *Inclination* of one Line to another; and you understand it of the *Inclination* of one Line from another, which is not *Inclination*, but *Declination*. For you make two straight Lines when they lye one on another, to lye *exilis* that is without any *Inclination* (because it serves your turn); not observing that it followeth thence that *Inclination* is a *digression* of one Line from another. This is in your first Argument.

Argument in the behalf of *Peletarius* (Pag. 10. Lin. 22.) and destroy his opinion. For according to *Euclide* the greatest Angle is the greatest Inclination, and an Angle equall to two Right Angles by this ἀλλοίωσις should not be the greatest Inclination, as it is, but the least that can be. But if by the Inclination of two Lines we understand that proceeding of them to a common Point which is caused by their generation, which (I believe) was *Euclides* meaning; then will the Angle of Contact be no less an Angle then a rectilineall Angle, but only (as *Clavius* truly saies it is) Heterogeneous to it; and the doctrine of *Clavius* more conformable to *Euclide* then that of *Peletarius*. Besides, if it be granted you, that there is no inclination of the Circumference to the Tangent, yet it does not follow that their concurrence doth not form some kind of Angle. For *Euclide* defineth there but one of the kinds of a Plain Angle. And then you may as much in vain seek for the Proportion of such an Angle to the Angle of Contact, as seek for the Focus, or Parameter of the Parabola of *Dives* and *Lazarus*. Your first argument therefore is nothing worth, except you make good that which in your second Argument you affirm, namely, That all Plain Angles, not excepting the Angle of Contact, are (Homogeneous) of the same kind. You prove it well enough of other Curvilineall Angles; but when you should prove the same of an Angle of Contact, you have nothing to say but Pag. 17. Lin. 15. *Unde autem illa quam somniet Heterogenia oriatur, neque potest ille ullatenus ostendere, neque ego vel somnare; whence should arise that diversity of kind, which he dreams of, neither can he at all shew, nor I dream; as if you knew what he could do if he were to answer you; or all were false which you cannot dream of.* So that besides your customary vanity, here is nothing hitherto proved neither for the opinion of *Peletarius*, nor against that of *Clavius*. I have I think sufficiently explicated in the first Lesson, That the Angle of Contact is Quantity, namely, that it is the Quantity of that crookedness or flexion, by which a straight Line is bent into an Arch of a Circle equall to it; and that because the crookedness of one Arch may be greater then the crookedness of another Arch of another Circle equall to it, therefore the Question *Quanta est curvitas*, How much is the crookedness, is pertinent, and to be answered by Quantity. And I have also shewn you in the same Lesson, that the Quantity of one Angle of Contact is compared with that of another Angle of Contact, by a Line drawn from the Point of Contact, and intercepted by their Circumferences; and that it cannot be compared by any measure with a Rectilineall Angle.

But let us see how you answer to that which *Clavius* has objected already. They are Heterogeneous, sayes he, because the Angle of Contact, how oft soever multiplied, can never exceed a Rectilineall Angle. To answer which you alledge, it is no Angle at all; and that therefore it is no Angle at all because the Lines have no Inclination one to another. How can Lines that have no Inclination one to another, ever come together? But you answer, at least they have no Inclination in the Point of Contact. And why have two straight Lines Inclination before they come to touch, more then a straight Line and an Arch of a Circle? And in the Point of Contact it self, how can it be that there is less Inclination of the two Points of a straight Line and an Arch of a Circle, then of the Points of two straight Lines? But the straight Lines you say will cut; Which is nothing to the Question; and yet this also is not so evident, but that it may receive an objection. Suppose two Circles A G B and C F B to touch in B, and have a common Tangent through B. Is not the Line C F B G A a crooked Line? And is it not cut by the common Tangent D B E? What is the Quantity of the two Angles F B E and G B D, seeing you say neither D B G nor E B F is an Angle? 'Tis not therefore the cutting of a crooked Line, and the touching of it, that distinguisheth an Angle simply, from an Angle of Contact. That which makes them differ, and in kind, is, that the one is the Quantity of a Revolution, and the other the Quantity of Flexion.

In the seventh Chapter of the same Treatise, you think you prove the Angle of Contact, if it be an Angle, and a Rectilineall Angle to



be (*Homogeneous*) of the same kind; (when you prove nothing but that you understand not what you say. Those Quantities which can be added together, or subtracted one from another, are of the same kind; But an Angle of Contact may be subtracted from a right Angle, and the Remainder will be the Angle of a Semicircle, &c. So you say, but prove it not, unless you think a man must grant you that the Superficies contained between the Tangent and the Arch, which is it you subtract, is the Angle of Contact; and that the Plain of the Semicircle is the Angle of the Semicircle, which is absurd; though as absurd as it is, you say it directly in your *Elenchus*, Pag. 35. Lin. 14. in these words, *When Euclide defines a Plain Angle to be the Inclination of two Lines, he meaneth not their aggregate, but that which lyes between them.* It is true, he meaneth not the aggregate of the two Lines; but that he means that which lyes between them, which is nothing else but an indeterminate Superficies, is false, or *Euclide* was as foolish a Geometrician as either of you two.

Again, you would prove the Angle of Contact, if it be an Angle, to be of the same kind with a Rectilineall Angle, out of *Euch.* 3. 16. Where he saies, *it is less then any acute Angle.* And it follows well, that if it be an Angle, and less then any Rectilineall Angle; it is also of the same kind with it. But to my understanding *Euclide* meant no more, but that it was neither greater nor equall; which is as truly said of Heterogeneous, as of Homogeneous Quantities. If he meant otherwise, he confirms the opinion of *Clavius* against you, or makes the Quantity of an Angle to be a Superficies, and indefinite. But I wonder how you dare venter to determine whether two Quantities be Homogeneous or not, without some Definition of Homogeneous (which is a hard word) that men may understand what it meaneth.

In your eighth Chapter you have nothing but Sir *H. Saviles* Authority, who had not then resolved what to hold; but esteeming the Angle of contact, first, as others falsely did, by the Superficies that lyes between the Tangent and the Arch, makes the Angle of Contact, and a Rectilineall Angle Homogeneous; and afterwards, because no multiplication of the Angle of Contact can make it equall to the least Rectilineall Angle, with great ingenuity returneth to his former uncertainty.

In your ninth and tenth Chapters you prove with much ado, that the Angles of like Segments are equal; as if that might not have been taken *gratis* by *Pelctarius* without Demonstration. And yet your Argument contained in the ninth Chapter is not a Demonstration, but a conjecturall discourse upon the word *Similitude*. And in the eleventh Chapter, wherein you answer to an objection, which might be made to your Argument in the precedent Page, taken from the Parallelism of two concentric Circles, though the objection be of no moment, yet you have in the same Treatise of yours that which is much more foolish, which is this, Pag. 38. Lin. 12. *Non enim magnitudo Anguli, &c. The magnitude of an Angle is not to be estimated by that straddling of the legs, which it hath without the Point of concurrence, but by that straddling which it hath in the Point of the concurrence it self.* I pray you tell me what straddling there is of two coincident Points, especially such Points as you say are nothing. When did you ever see two nothings straddle?

The Arguments in your twelfth and thirteenth Chapters are grounded all on this untruth, that an Angle is that which is contained between the Lines that make it, that is to say, is a Plain Superficies. Which is manifestly false; because the measure of an Angle is an Arch of a Circle, that is to say, a Line; which is no measure of a Superficies. Besides this gross ignorance, your way of Demonstration by putting *N* for a great Number of sides of an equilateral Polygon, is not to be admitted. For though you understand something by it, you demonstrate nothing to any Body, but those who understand your Symbolique tongue, which is a very narrow Language. If you had demonstrated it in Irish, or Welsh, though I had not read it, yet I should not have blamed you, because you had written to a considerable Number of mankind, which now you do not.

In your last Chapters you defend *Vicellio* without need; for there is no doubt but that whatsoever crooked Line be touched by a straight Line, the Angle of Contingence will neither add any

any thing to, nor take any thing from a Rectilineall Right Angle; but that it is because the Angle of Contact is no Angle, or no Quantity, is not true. For it is therefore an Angle, because an Angle of Contact; and therefore Quantity, because one Angle of Contact may be greater then another; and therefore Heterogeneall, because the measure of an Angle of Contact cannot (congruere) be applyed to the measure of a Rectilineall Angle, as they think it may, who affirm with you that the Nature of an Angle consisteth in that which is contained between the Lines that comprehend it, viz. in a plain Superficies. And thus you see in how few Lines, and without Brachygraphie, your Treatise of the Angle of Contingence is discovered for the greatest part to be false, and for the rest, nothing but a detection of some errors of *Clavius* grounded on the same false Principles with your own. To return now from your Treatise of the Angle of Contact back again to your *Elenchus*.

The fault you find at Art. 18. is, that I understand not that *Euclide* makes a Plain Angle to be that which is contained between the two Lines that form it. 'Tis true, that I do not understand that *Euclide* was so absurd, as to think the nature of an Angle to consist in Superficies; but I understand that you have not had the wit to understand *Euclide*.

The nineteenth Article of mine in this fourteenth Chapter is this; *All respect, or variety of Position of two Lines, seemeth to be comprehended in four kinds. For they are either Parallel; or, (being if need be produced) make an Angle; or, (if drawn out faire enough) Touch; or lastly, they are Asymptotes.* In which you are first offended with the word *It seems*. But I allow you that never erre, to be more peremptory then I am. For to me it seemed, I say again seemed, that such a Phrase, in case I should leave out something in the enumeration of the severall kinds of Position, would save me from being censured for untutth. And yet your instance of two straight Lines in divers Plains, does not make my enumeration insufficient. For those Lines though not Parallels, nor cutting both the Plains, yet being moved Parallely from one Plain to another, will fall into one or other of the kinds of position by me enumerated; and consequently are as much that position, as two straight Lines in the same Plain not parallel, make the same Angle, though not produced till they meet, which they would make if they were so produced. For you have no where proved, nor can prove, that two such Lines do not make an Angle. It is not the actuall concurrence of the Lines, but the Arch of a Circle, drawn upon that point for Center, in which they would meet, if they were produced, and intercepted between them, that constitutes the Angle.

Also your objection concernin' Asymptotes in generall, is absurd. You would have me add, that their distance shall at last be less then any distance that can be assigned; and so make the definition of the *Genus* the same with that of the *Species*. But because you are not Professor of Logick, it is not necessary for me to follow your counsell. In like manner, if we understand one Line to be moved towards another always parallelly to itself, which is, though not actually, yet potentially the same position, all the rest of your instances will come to nothing.

At the two and twentieth Article you object to me the use of the word *Figure*, before I had defined it: wherein also you do absurdly; for I have no where before made such use of the word *Figure*, as to argue any thing from it; and therefore your objection is just as wise as if you had found fault with putting the word *Figure* in the Titles of the Chapters placed before the Book. If you had known the nature of Demonstration, you had not objected this.

You add further, that by my Definition of *Figure*, a solid Sphere, and a Sphere made hollow within, is the same Figure; but you say not why, nor can you derive any such thing from my definition. That which deceived your shallowness, is, that you take those Points that are in the concave Superficies of a hollowed Sphere, not to be contiguous to any thing without it, because that whole concave Superficies is within the whole Sphere. Lastly, for the fault you find, with the definition of *like Figures in like positions*; I confesse there wants the same word which was wanting in the Definition of Parallels, namely, *ad eadem partes* (the same way) which should have been added in the end of the definition of like Figures, &c. and may easily be supplied by any student of Geometry, that is not otherwise a fool.

At the fifteenth Chapter Art. 1. Numb. 6. you object as a contradiction, that *I make Motion to be the measure of Time*, and yet in other places do usually measure Motion and the affections thereof by Time. If your thoughts were your own, and not taken rashly out of Books, you could not but (with all men else that see Time measured by Clocks, Dyals, Hour-glasses, and the like) have conceived sufficiently, that there cannot be of Time any other measure besides Motion; and that the most universall measure of Motion, is a Line described by some other Motion. Which Line being once exposed to sense, and the motion whereby it was described sufficiently explicated, will serve to measure all other Motions and their Time; for Time and Motion (Time being but the mentall Image or remembrance of the motion) have but one and the same dimension, which is a Line. But you that would have me measure *swiftness and slowness* by longer and shorter motion, what do you mean by *longer and shorter motion*? Is *longer and shorter*, in the motion, or in the Duration of the motion, which is Time? Or is the Motion, or the Duration of the motion that which is exposed, or designated by a Line? Geometricians say often, *let the Line AB, be the Time*; but never say, *let the Line AB be the Motion*. There is no unlearned man that understandeth not what is Time, and Motion, and Measure; and yet you that undertake to teach it (most egregious Professors) understand it not.

At the second Article you bring another Argument (which it seems in its proper place, you had forgotten) to prove that a Point is not Quantity nor considered, but absolutely Nothing; which is this, *That if a Point be not nothing, then the whole is greater then its two halves*. How does that follow? Is it impossible when a Line is divided into two halves that the middle Point should be divided into two halves also, being Quantity?

At the seventh Article, I have sufficiently demonstrated, that all Motion is infinitely propagated, as far as space is filled with Body. You alleadge no fault in the demonstration, but object from sense, *that the skipping of a Flea, is not propagated to the Indies*. If I ask you how you know it, you may wonder perhaps; but answer you cannot. Are you Philosophers or Geometricians, or Logicians, more then are the simplest of rurall people? Or are you not rather less, by as much as he that standeth still in ignorance, is nearer to knowledge, then he that runneth from it by erroneous learning?

And lastly, what an absurd objection is it which you make to the eighth Article, where I say that *when two Bodies of equal magnitude fall upon a third Body, that which falls with greater velocity, imprints the greater motion*? You object, *that not so much the magnitude is to be considered as the weight*; as if the weight made no difference in the velocity, when notwithstanding weight is nothing else but motion downward? Tell me, when a weighty body thrown upwards worketh on the Body it meeteth with, do you not then think it worketh the more for the greatnes, and the less for the weight?

Of

Of the Faults that Occurre in Demonstration.

To the same egregious Professors of the Mathematicks in the Univerſity of Oxford.

LESSON IIII.

OF twenty Articles which you ſay (of nineteen which I ſay) make the ſixteenth Chapter; you except but three, and confidently affirm the reſt are falſe. On the contrary, except three or four faults, ſuch as any Geometrician may ſee proceed not from ignorance of the Subject, or from want of the Art of Demonſtration, (and ſuch as any man might have mended of himſelf) but from ſecurity; I affirm that they are all true, and truly Demonſtrated; and that all your objections proceed from meer ignorance of the Mathematicques.

The firſt fault you find is this, that I expreſs not, (*Art. 1.*) what *Impetus* it is, which I would have to be multiplied into the Time.

The laſt Article of my thirteenth Chapter was this, *If there be a Number of Quantities propounded, howſoever equall or unequall to one another; and there be another Quantity which ſo often taken as there be Quantities propounded, is equall to their whole ſum; that Quantity I call the mean Arithmetically of them all.* Which Definition I did there inſert to ſerve me in the explication of thoſe Propoſitions of which the ſixteenth Chapter conſiſteth, but did not uſe it here as I intended. My firſt Propoſition therefore as it ſtandeth yet in the Latine, being this, *The velocity of any Body moved during any Time, is ſo much as is the product of the Impetus in one Point of Time, multiplied into the whole Time; to a man that hath not ſkill enough to ſupply what is wanting, is not intelligible.* Therefore I have cauſed it in the Engliſh to go thus, *The velocity of any Body in whatſoever Time moved, hath its Quantity determined by the ſum of all the ſeverall (Impetus) Quickneſſes, which it hath in the ſeverall Points of the Time of the Bodies motion.* And added, *that all the Impetus together taken through the whole Time is the ſame thing with the Mean Impetus (which Mean is defined Chapter 13. Art. 29.) multiplied into the whole Time.* To this firſt Article, as it is uncorrected in the Latine, you object, *That meaning by Impetus ſome middle Impetus, and aſſigning none, I determine nothing.* And 'tis true. But if you had been Geometricians ſufficient to be Profeſſors, you would have ſhewed your ſkill much better, by making it appear that this middle Impetus could be none but that, which being taken ſo often, as there be Points in the Line of Time, would be equall to the ſum of all the ſeverall Impetus taken in the Points of Time reſpectively; which you could not do.

To the Corollary, you aſk firſt how *Impetus* can be ordinately applied to a Line; Abſurdly. For does not *Archimedes* ſometimes ſay, and with him many other excellent Geometricians, *let ſuch a Line be the Time?* And do they not mean, that that Line, or the motion over it, is the

the measure of the Time? And may not also a Line serve to measure the swiftness of a Motion? You thought (you say) only Lines ought to be said to be ordinately applied to Lines. Which I easily believe; for I see you understand not that a Line, though it be not the Time it self, may be the quantity of a Time. You thought also all you have, said in your *Elenchus*, in your Doctrine of the Angle of Contact, in your *Arithmetica Infinitorum*, and in your *Coniques* is true; and yet it is almost all proved false, and the rest nothing worth.

Secondly, you object, that I design a *Parallelogram* by one only side. It was indeed a great oversight, and argueth somewhat against the man, but nothing against his Art. For he is not worthy to be thought a Geometrician that cannot supply such a fault as that, and correct his Book himself. Though you could not do it, yet another from beyond Sea took notice of the same fault in this manner; He makes a *Parallelogram* of but one side; it should be thus, *Vel denique per parallelogrammum cuius unum latus est medium proportionale inter Impetum maximum (five ultimò acquisitum) & impetum ejusdem maximi semissem; alterum vero latus, medium proportionale, inter totum tempus, & ejusdem totius temporis semissem.* Which I therefore repeat, that you may learn good manners; and know, that they who reprehend, ought also, when they can, to add to their reprehension the correction.

At the second Article, you are pleased to advise me, instead of *In omni motu uniformi*, to put in *In omnibus motibus uniformibus*. You have a strange opinion of your own Judgement, to think you know to what end another man useth any word, better then himself. My intention was only to consider motions uniform, and motions from rest uniformly, or, regularly accelerated, that I might thereby compute the lengths of crooked Lines, such as are described by any of those motions. And therefore it was enough to prove this Theorem to be true in all uniform or uniformly accelerated Motion, not Motions; though it be true also in the Plurall. It seems you think a man must write all he knows, whether it conduce, or not, to his intended purpose. But that you may know that I was not, (as you think) ignorant how far it might be extended, you may read it Demonstrated at the same Article in the English universally. Against the demonstration it self you run to another Article, namely, the thirteenth, which is this Problem; *The length being given, which is passed over in a given Time by uniform Motion, to find the length which shall be passed over by Motion uniformly accelerated in the same Time, so as that the Impetus last acquired be equal to the Time.* Which you recite imperfectly, thereby to make it seem that such a Length is not determined. Whether you did this out of ignorance, or on purpose, thinking it a piece of wit, as your pretended mysterie which goes immediately before, I cannot tell, for in neither place can any wit be espied by any but your selves. To imagine Motions with their Times and Wayes, is a new business, and requires a steady brain, and a man that can constantly read his own thoughts, without being diverted by the noise of words. The want of this ability, made you stumble and fall unhand somely in the very first place, (that is in Chap. 13. Art. 13.) where you venture to reckon both Motion and Time at once; and hath made you in this Chapter to stumble in the like manner at every step you go. As for example, when I say, *as the product of the Time, and Impetus, so the product of the Time and Impetus, to the Space to the Space when the Motion is Uniform*, you come in with nay, rather as the Time to the Time; as if the *Parallelograms* A I, and A H, were not also as the Times A B, and A F. Thus it is, when men venture upon ways they never had been in before, without a guide.

In the Corollary, you are offended with the permutation of the Proportion of Times and Lines, because you think, you that have scarce one right thought of the Principles of Geometry, that Line, and Time are Heterogeneous Quantities. I know Time and Line are of divers natures; and more, that neither of them is Quantity. Yet they may be both of them *Quanta*, that is, they may have Quantity; but that their Quantities are Heterogeneous is false. For they are compared and measured both of them by straight lines. And to this there is nothing contrary in the place cited by you out of *Cleuius*; or if there were, 'twere not to be valued. And to your question what is the Proportion of an Hour to an Ell; I answer, it is the same Proportion

Proportion that *two Hours* have to *two Ells*. You see your Question is not so subtle as you thought it: By and by you confess that in Times and Lines there is *Quid Homogeneum* (this *Quid* is an infallible sign of not fully understanding what you say) which is false if you take it of the Lines themselves; though if you take it of their Quantities, it is true without a *Quid*. Lastly, you tell me how I might have expressed my self so as it might have been true. But because your expressions please me not, I have not followed your advice.

To the third Article, which is this, *In motu uniformiter à quiete accelerato, &c. In motion uniformly accelerated from Rest, that is, when the Impetus increaseth in Proportion to the Times, The Length run over in one Time is to the Length run over in another Time, as the Product of the Impetus multiplied by the Time, to the Product of the Impetus multiplied by the Time;* you object, that the Lengths run over, are in that Proportion which the Impetus hath, to the Impetus; not that which the Impetus hath to the Time, because Impetus to Time has no Proportion, as being Heterogeneous. First, when you say the Impetus, do you mean some one Impetus designed by some one of the unequall straight Lines Parallel to the Base *BI*? That is manifestly false. You mean the aggregate of all those unequall Parallels. But that is the same thing with the Time multiplied into the mean Impetus. And so you say the same that I do. Again, I ask where it is that I say or dream that the Lengths run over are in the Proportion of the Impetus to the Times? Is it you or I that dream? And for your Heterogeneity of the Quantities of Time and of Swiftness, I have already in divers places shewed you your error. Again, Why do you make *BI* represent the Lengths run over, which I make to be represented by *DE*, a Line taken at pleasure; and you also a few Lines before make the same *BI* to design the greatest acquired Impetus? These are things which shew that you are puzzled and intangled with the unusuall speculation of Time and Motion, and yet are thrust on with Pride and Spite to adventure upon the examination of this Chapter.

Secondly, you grant the Demonstration to be good, supposing I meane it (as I seeme to speak) of one and the same Motion. But why doe I not meane it of one and the same Motion, when I say not in *Motions*, but in *Motion* uniform? Because (say you) in that which follows, I draw it also to different Motions. You should have given at least one Instance of it; but there is no such matter. And yet the Proposition, is in that case also true; though then it must not be Demonstrated by the similitude of Triangles, as in the case present. And therefore the objections you make from different Impetus acquired in the same time, and from other cases which you mention, are nothing worth.

At the fourth Article, you allow the Demonstration all the way (except the faults of the third, which I have already proved to be none) till I come to say, that because the Proportion of *FK* to *BI* is double to the Proportion of *AF* to *AB*, therefore the Proportion of *AB* to *AF* is double to the Proportion of *BI* to *FK*. This you deny, and wonder at, as strange (for it is indeed strange to you) and in many places you exclaim against it as extream Ignorance in Geometry. In this place you onely say, *No such matter; for though one Proportion be double to another, yet it does not follow that the Converse is the double of the Converse*. So that this is the issue to which the Question is reduced, whether you have any or no Geometry. I say, if there be three Quantities in continuall Proportion, and the first be the least, the Proportion of the first to the second is double to the Proportion of the first to the third; and you deny it. The reason of our dissent consisteth in this, that you think the doubling of a Proportion to be the doubling of the Quantity of the Proportion, as well in Proportions of Defect, as in Proportions of Excess; and I think that the doubling of a Proportion of Defect, is the doubling of the defect of the Quantity of the same. As for example in these three numbers, 1, 2, 4, which are in continuall Proportion, I say the Quantity of the Proportion of one to two, is double the Quantity of the Proportion of one to four. And the Quantity of the Proportion of one to four, is half the Quantity of the Proportion of one to two. And yet deny not but that the Quantity of the Defect in the Proportion of one to two is doubled in the Proportion of one to four. But because the doubling of defect makes greater defect, it

maketh the Quantity of the Proportion less. And as for the part which I hold in this Question, first, there is thus much demonstrated by *Euclide*, El. 5. Prop. 8; that the Proportion of one to two is greater than the Proportion of one to four, though how much it is greater be not there Demonstrated. Secondly, I have Demonstrated (Chap. 13. Art. 16.) That it is twice as great, that is to say, (to a man that speaks English) double. The introducing of *duplicate*, *triplicate*, &c. instead of *double*, *triple*, &c. (though now they be words well understood by such as understand what Proportion is) proceeded at first from such as durst not for fear of absurdity, call the half of any thing double to the whole, though it be manifest that the half of any defect is a double Quantity to the whole defect; for want added to want maketh greater want, that is, a less positive Quantity. This difference between *double*, and *duplicate*, lighting upon weak understandings, has put men out of the way of true reasoning in very many Questions of Geometry. *Euclide* never used but one word both for *double* and *duplicate*. It is the same fault when men call half a Quantity *Subduplicate*, and a third part *Subtriplicate* of the whole, with intention (as in this case) to make them pass for words of signification different from the *half* and the *third part*. Besides, from my Definition of Proportion (which is clear, and easie to be understood by all men, but such as have read the Geometry of others unluckily) I can Demonstrate the same evidently and briefly thus. My Definition is this, *Proportion is the Quantity of one Magnitude taken comparatively to another*. Let there be therefore three Quantities, 1, 2, 4, in continuall Proportion. Seeing therefore the Quantity of four in respect of one, is twice as great as the Quantity of the same four in respect of 2, it followeth manifestly that the Quantity of 1 in respect of 4, is twice as little as the Quantity of the same 1 in respect of 2; and consequently the Quantity of one in respect of two, is twice as great as the Quantity of the same one in respect of four; which is the thing I maintain in this Question. Would not a man that employes his time at Bowles, choose rather to have the advantage given him of three in nine, then of one in nine? And why, but that three is a greater Quantity in respect of nine, then is one? Which is as much as to say, three to nine hath a greater Proportion then one to nine; as is Demonstrated by *Euclide*, El. 5.3. Is it not therefore (you that profess Mathematicques, and Theology, and practise the depression of the truth in both) well ow'd of you, to teach the contrary? But where you say that the Point K (in the second Figure of the Table belonging to this 16. Chapter) is not in the Parabolicall Line whose Diameter is A B, and Base B I, but in the Parabolicall Line of the Complement of my Semiparabola (as I may learn from the twenty-third Proposition of your *Arithmetica Infinitorum*) whose Diameter is A C, and Base I C. What Line is that? Is it the same Line with that of my Semiparabola, or not the same? If the same, why find you fault? If not the same, you ought to have made a Semiparabola on the Diameter A C, and Base I C, and following my Constitution made it appear that K is not in the Line wherein I say it is; which you have not done, nor could do.

Then again, running on in the same blindness of Passion, you pretend I make the Proportion of B I to F K double to that of A B to A F, and then confute it; when you knew I made the Proportion of F K to B I, double to that of F N, to B I, that is, of A F to A B; and this was it you should have confuted. That which followeth is but a Triumphant in your own Ignorance, wherein you also say, *That all that I afterwards build upon this Doctrine is false*. You see whether it be like to prove so or not. As for your *Arithmetica Infinitorum*, I shall then read to you a piece of a Lesson on it when I come to your objections against the next Chapter. In the mean Time let me tell you, it is not likely you should be great Geometricians, that know not what is Quantity, nor Measure, nor Straight, nor Angle, nor Homogeneous, nor Heterogeneous, nor Proportion, as I have already made appear in this and the former Lessons.

To the first Corollary of this fourth Article your exception I confess is just, and (which I wonder at) without any incivility. But this argues not Ignorance, but Security. For who is there that ever read any thing in the Coniques, that knows not that the parts of a Parabola cut

cut off by Lines Parallel to the Base, are in Triplicate Proportion to their Bases? But having hitherto designed the Time by the Diameter, and the *Impetus* by the Bas. ; and in the next Chapter (where I was to calculate the Proportion of the Parabola, to the Parallelogram) intending to design the Time by the Base, I mistook and put the Diameter again for the Time; which any man but you might as easily have corrected as reprehended.

/ To the second *Corollarie*, which is this, *That the Lengths run over in equall Times by Motion so accelerated, as that the Imperus increase in double Proportion to their Times, are as the differences of the Cubick numbers beginning at unity, that is, as seven, nineteen, thirty-seven, &c.* You say 'tis false. But why? Because (say you) portions of the Parabola of equall altitude taken from the beginning are not as those numbers seven, nineteen, thirty-seven, &c. Does this think you, contradict any thing in this Proposition of mine? Yes, because (you think) the lengths gone over in equall Times, are the same with the parts of the Diameter cut off from the Vertex, and proportionall to the numbers one, two, three, &c. Whereas the lengths run over, are as the aggregates of their velocities, that is, as the parts of the Parabola itself, that is, as the Cubes of their Bases, that is, as the numbers one, eight, twenty-seven, sixty-four, &c. and consequently the lengths run over in equall Times, are as the differences of those Cubick Numbers one, eight, twenty-seven, sixty-four, whose differences are seven, nineteen, thirty-seven, &c. The cause of your mistake was, that you cannot yet, nor perhaps ever will contemplate Time and Motion (which requireth a steady brain) without confusion.

The third *Corollary*, you also say is false, *Whether it be meant of Motion uniformly accelerated (as the words are) or (as perhaps, you say, I meant it) of such Motion as is accelerated in double Proportion to the Times.* You need not say perhaps I meant it. The words of the Proposition are enough to make the meaning of the *Corollary* understood. But so also you say it is false. Me thinks you should have offered some little proof to make it seem so. You think your Authority will carry it. But on the contrary I believe rather that they that shall see how your other objections hitherto have sped, will the rather think it true, because you think it false. The Demonstration as it is, is evident enough; and therefore I saw no cause to change a word of it.

To the fifth Article you object nothing, but that it dependeth on this Proposition (Chap. 13. Art. 16.) *That when three Quantities are in continuall Proportion, and the first is the least, as in these numbers, four, six, nine. The Proportion of the first to the second is double to the Proportion of the same first to the last.* Which is there demonstrated, and in the former Lessons so amply explicated, as no man can make any further doubt of the truth of it. And you will, I doubt not assent unto it. But in what estate of mind will you be then? A man of a tender forehead after so much insolence, and so much contumelious language grounded upon arrogance and ignorance, would hardly indure to out-live it. In this vanity of yours, you ask me whether I be angry, or blush, or can endure to hear you. I have some reason to be angry; for what man can be so patient as not to be moved with so many injuries? And I have some reason to blush, considering the opinion men will have beyond Sea, (when they shall see this in Latine) of the Geometry taught in Oxford. But to read the worst you can say against me, I can indure, as easily at least, as to read any thing you have written in your Treatises of the *Angle of Contact*, of the *Conique-sections*, or your *Arithmetica Infinitorum*.

The sixth, seventh, eighth Articles, you say are sound. True. But never the more to be thought so for your approbation, but the less; because you are not fit, neither to reprehend, nor praise; and because all that you have hitherto condemned as false, hath been proved true. Then you shew me how you could demonstrate the sixth and seventh Articles a shorter way. But though there be your Syniboles, yet no man is obliged to take them for demonstration. And though they be granted to be dumb Demonstrations, yet when they are taught to speak as they ought to do, they will be longer Demonstrations than these of mine.

To the ninth Article, which is this, *If a Body be moved by two Movents at once, concurring in what Angle soever, of which, one is moved uniformly, the other, with Motion uniformly*

formly accelerated from Rest, till it acquire an Impetus equal to that of the uniform motion, the line in which the Body is carried, shall be the crooked line of a Semiparabola. You lift up your voice again, and ask what Latitude? what Diameter? what Inclination of the Diameter to the Ordinate Lines? If your Founder should see this, or the like objections of yours; he would think his money ill bestowed. When I say in what Angle forever, you ask, in what Angle? When I say two Movements, one uniform, the other uniformly accelerated, make the Body describe a Semiparabolline; you ask, which is the Diameter; as not knowing that the accelerated motion describes the Diameter, and the other a Parallel to the Base. And when I say the two Movements meet in a Point, from which Point both the Motions begin, and one of them from Rest, you ask me, what is the Altitude? As if that Point where the Motion begins from Rest were not the Vertex; or that the Vertex and Base being given, you had not wit enough to see that the Altitude of the Parabola is determined? When Galileo's Proposition, which is the same with this of mine, supposed no more but a Body moved by these two Motions, to prove the Line described to be the crooked Line of a Semiparabola, I never thought of asking him what Altitude, nor what Diameter, nor what Angle, nor what Base had his Parabola. And when Archimedes said, let the Line A B be the Time, I should never have said to him, do you think Time to be a Line, as you ask me whether I think Impetus can be the Base of a Parabola. And why, but because I am not so egregious a Mathematician, as you are. In this giddiness of yours, caused by looking upon this intricate business of Motion, and of Time, and the concurrence of Motion uniform, and uniformly accelerated, you rave upon the numbers 1, 4, 9, 16, &c. without reference to any thing that I had said; insomuch as any one that had seen how much you have been deceived in them before in your scurvy Book of *Arithmetica Infinitorum*, would presently conclude, that this objection was nothing else but a fit of the same madness which posselt you there.

My tenth Article is like my ninth; and your objections to it are the same which are to the former. Therefore you must take for answer just the same which I have given to your objection there.

To the eleventh, you say first, you have done it better at the sixty-fourth Article of your *Arithmetica Infinitorum*. But what you have done there, shall be examined when I come to the Defence of my next Chapter. And whereas I direct the Reader for the finding of the Proportions of the Complements of those Figures to the Figures themselves, to the Table of Art. 3. Chap. 17. you say that if the increase of the Spaces, were to the increase of the Times, as one to two, then the complement should be to the Parallelogram as one to three, and say you find not $\frac{1}{3}$ in the Table. Did you not see that the Table is only of those Figures which are described by the concurrence of a Motion uniform with a motion accelerated? You had no reason there-

fore to look for $\frac{1}{3}$ in that Table; for your case is of Motion uniform concurring with Motion retarded, because you make not the Proportions of the Spaces to the Proportions of the Times as two to one, but the contrary; so that your objection ariseth from want of observing what you read. But I may learn (you say) these, and greater Matters than these, in your twenty-third and sixty-fourth Propositions of your *Arithmetica Infinitorum*. This which you say here is a great Absurdity; but if you mean I shall finde greater there, I will not say against you. This $\frac{1}{3}$ you looked for, belongs to the Complements of the Figures calculated in that Table; which

because you are not able to find out of your selves, I will direct you to them. Your case is of $\frac{1}{3}$ for the Complement of a Parabola. Take the Denominator of the Fraction which belongs to the Parabola, namely three, and for Numerator take the Numerator of the Fraction which belongs to the Triangle, namely one, and you have the Fraction sought. And in like manner for the Complements

Complement of any other Figure. As for example, of the second Parabola, whose Fraction hath for Denominator five, take the Numerator of the Fraction of the same Triangle which is one, and you have $\frac{1}{5}$ for the Fraction sought for; and so of the rest, taking alwayes one for the Numerator.

The twelfth Article, which you say is miserably false, I have left standing unaltered. For not comprehending the sense of the Proposition, you make a Figure of your own, and fight against your own fancied Motions, different from mine. Other Geometricians that understand the construction better, find no fault. And if you had in your own fifth Figure drawn a Line through N Parallel to A E, and upon that Line supposed your accelerated Motion, you would quickly have seen that in the Time A E, the Body moved from Rest in A, would have fallen short of the Diagonal A D; and that all your extravagant pursuing of your own mistake had been absurd.

My thirteenth Article you say is ridiculous. But why? *The Impetus last acquired cannot (you say) be equal to a Time.* But the Quantity of the Impetus may be equal to the Quantity of a Time, seeing they are both measured by Line. And when they are measured by the same described Line, each of their Quantities is equal to that same Line, and consequently to one another. But when I meet with this kind of objection again, since I have so often already shewn it to be frivolous, and no less to be objected against all the Antients that ever Demonstrated any thing by Motion, then against me, I purpose to neglect it.

Secondly, you object *That Motion uniformly accelerated does no more determine Swiftness, then Motion uniform.* True; you needed not have used sixteen Lines to set down that. But suppose I add (as I do) so as the last acquired Impetus be equal to the Time. But that (you say) is not sense; Which is the objection I am to neglect. But (you say again) supposing it sense, this limitation helps me nothing. Why? *Because (you say) a Parabola may be described upon a Base given, and yet have any Altitude, or any Diameter one will.* Who doubts it? But how follows it from thence, that when a Parabolicall Line is described by two Motions, one uniform, the other uniformly accelerated from Rest, that the determining of the Base does not also determine the whole Parabola? But fifthly (you say) *that this equality of the Impetus to the Time does not determine the Base.* Why not? *Because (you say) it is an error proceeding from this, that I understand not what is Ratio subduplicata.* I look't for this. I have shewn and inculcated sufficiently before, that the error is on your side; and therefore must tell you, that this objection, and also a great part of the rest of your errors in Geometry proceedeth from this, that you know not what Proportion is. But see how wisely you argue about this duplication of Proportion. For thus you say *verbatim.* Stay a little. *What Proportion has duplicate Proportion to single Proportion? Is it alwayes the same? I think not. For example. Duplicate Proportion $\frac{4}{1} = \frac{2}{1}$ in $\frac{2}{1}$ is double to the single $\frac{2}{1}$. Duplicate Proportion $\frac{9}{1} = \frac{3}{1}$ in $\frac{3}{1}$ is triple to its single $\frac{3}{1}$.* Let any man, even of them that are most ready in

your Symboles, say in your behalf (if he be not ashamed) that the Proportion of nine to one is triple to the Proportion of three to one, as you do.

In the fourteenth, fifteenth, and sixteenth Article, you bid me repeat your objections to the thirteenth. I have done it; and find that what you have objected to the thirteenth, may as well be objected to these; and consequently, that my answer there will also serve me here. Therefore (if you can endure it) read the same answer over again.

But you have not yet done (you say) with these Articles. Therefore (after you had for a while spoken perplexity, conjecturing not without just cause, that I could not understand you) you say that to the end I may the better perceive your meaning, I should take the example following. *Let a Motion (in the first Figure of this Chapter) be moved uniformly in the Time A B, with the continuall Impetus A C, or B I, whose whole velocity shall therefore be the*

Paral-

Parallelogram $ACIB$. And another Movent be uniformly accelerated, so as in the Time AB it acquire the same Impetus BI . Now as the whole velocity, is to the whole velocity, so is the length run over, to the length run over. All this I acknowledge to be according to my sense, laying that your putting your word *Movens* instead of my word *Mobile* hath corrupted this Article. For in the first Article, I meddle not with Motion by Concourse, wherein only I have to do with two Movents to make one Motion; but in this I do, wherein my word is not *Movens* but *Mobile*; by which it is easie to perceive you understand not this Proposition. Then you proceed, *But the length run over by that accelerated Motion is greater then the length run over by that uniform Motion.* Where do I say that? You answer, in the ninth and thirteenth Article, in making AC (in the fifth Figure) greater then AB ; and AH (in the eighth Figure) greater then AB ; and consequently, the Triangle ABI , greater then the Parallelogram $ACIB$. That consequently is without consequence; for it importeth nothing at all in this Demonstration, whether AB , or AC in the fifth Figure be the greater. Besides, I speak there of the Concourse of two Movents that describe the Parabolicall Line AGD ; where the increasing Impetus, (because it increaseth as the Times) will be designed by the Ordinate Lines in the Parabola $AGDB$. And if both the Motions in AB and AC were uniform, the Aggregate of the Impetus would be designed by the Triangle ABD , which is less then the Parallelogram $ACDB$. But you thought that the Motion made by AC uniformly, is the same with the Motion made uniformly in the same Time by the Motions in AB , and AC concurring. So likewise in the eighth Figure, there is nothing hinders AH from being greater then AB , unless I had said that AB had been described in the Time AC with the whole Impetus AC maintained entire; of which there is nothing in the Proposition, nor would at all have been pertinent to it. Therefore all this new undertaking of the thirteenth, fourteenth, fifteenth and sixteenth Articles, is to as little purpose as your former objections. But I perceive that these new and hard Speculations, though they turn the edge of your wit, turn not the edge of your malice.

At the seventeenth Article, you shew again the same confusion. Return to the eighth Figure, *If in a Time given a Body run over two lengths, one with uniform, the other with accelerated Motion;* as for example, if in the same Time AC a Body run over the Line AB with uniform Motion, and the Line AH with Motion accelerated; and again in a part of that Time is run over a part of the length AH , with uniform Motion, and another part of the same with Motion accelerated; as for example, in the Time AM it run over with uniform Motion the Line AI , and with Motion accelerated the Line AB . I say the excess of the whole AH above the part AI , is to the excess of the whole AB above the part AI , as the whole AH , to the whole AB . But first you will say, that these words *as the whole AH to the whole AB* , are left out in the Proposition. But you acknowledge that it was my meaning; and you see it is expressed before I come to the Demonstration. And therefore it was absurdly done to reprehend it. Let us therefore pass to the Demonstration. Draw IK Parallel to AC , and make up the Parallelogram $AIKM$. And supposing first the acceleration to be uniform, divide IK in the midst at N ; and between IN , and IK , take a mean Proportionall IL . And the straight Line AL , drawn and produced, shall cut the Line BD in F , and the Line CG in G (which Lines CG , and BD , as also HG and BE , are determined (though you could not carry it so long in memory) by the Demonstration of the thirteenth Article.) For seeing AB is described by Motion uniformly accelerated, and AI by Motion uniform in the same Time AM ; and IL is a mean Proportionall between IN (the half of IK) and IK ; therefore by the Demonstration of the thirteenth Article; AI is a mean Proportionall between AB and the half of AB , namely AO . Again, because AB is described by uniform Motion, and AH by Motion uniformly accelerated, both of them in the same Time AC , BE is a mean Proportionall between BD and half BD , namely BE ; Therefore by the Demonstration of the same thirteenth Article, the straight Line ALF produced will fall on G ; and the Line AH will be to the Line AB , as the Line AB to the Line AL . And consequently as AH to AB ,

§ HB to BI; which was to be Demonstrated. And by the like Demonstration the same may be proved, where the acceleration is in any other Proportion that can be assigned in Numbers, saving that whereas this Demonstration dependeth on the construction of the thirteenth Article, if the Motion had been accelerated in double Proportion to the Times, it would have depended on the fourteenth, where the Lines are determined. Which determinations being not repeated, but declared before in the thirteenth Article, to which this Diagram belongeth, you take no notice of, but go back to a Figure belonging to another Article where there was no use of these Determinations. But because I see that the words of the Proposition, are as of four Motions, and not of two Motions made by twice two Movents, I must pardon them that have not rightly understood my meaning; and I have now made the Proposition according to the Demonstration. Which being done, all that you have said in very neer two leaves of your *Elenchus* comes to nothing; and the fault you find comes to no more then a too much trusting to the skill and diligence of the Reader. And whereas after you had sufficiently troubled your self upon this occasion, you add, *that if Sir H. Savile had read my Geometry, he had never given that censure of Joseph Scaliger, in his Lectures upon Euclide, that he was the worst Geometrician of all Mortall men, not excepting so much as Orontius, but that praise should have been kept for me*; You see by this time, at least others do, how little I ought to value that opinion; and that though I be the least of Geometricians, yet my Geometry is to yours as 1 to 0. I recite these words of yours, to let the world see your indiscretion in mentioning so needlessly that passage of your Founder. It is well known that *Joseph Scaliger* deserved as well of the state of Learning, as any man before or since him; and that though he failed in his Ratiocination concerning the Quadrature of the Circle, yet there appears in that very failing so much knowledge of Geometry, that *Sir H. Savile* could not but see that there were mortall men very many that had less; and consequently he knew that that censure of his in a rigid sense (without the License of an *Hyperbole*) was unjust. But who is there that will approve of such *Hyperboles* to the dishonour of any but of unworthy persons, or think *Joseph Scaliger* unworthy of honour from Learned men? Besides, it was not *Sir H. Savile* that confused that false Quadrature, but *Clavius*. What honour was it then for him to triumph in the victory of another? When a beast is slain by a Lion, is it not easie for any of the Fowles of the Air to settle upon, and peck him? Lastly, though it were a great error in *Scaliger*, yet it was not so great a fault as the least Sin; and I believe that a publique contumely done to any worthy person after his death, is not the least of Sins. Judge therefore whether you have not done indiscreetly, in reviving the onely fault, perhaps that any man living can lay to your Founders charge; and yet this error of *Scaligers* was no greater then one of your own of the like nature, in making the true Spirall of Archimedes equall to half the circumference of the Circle of the first revolution; and then thinking to cover your fault by calling it afterwards an Aggregate of Arches of Circles (which is no Spirall at all of any kind) you do not repair but double the absurdity. What would *Sir Henry Savile* have said to this?

The eighteenth Article is this, *In any Parallelogram, if the two sides that contain the Angle be moved to their opposite sides, the one uniformly, the other uniformly accelerated; the side that is moved uniformly, by its concurrence through all its Longitude, hath the same effect which it would have if the other Motion were also uniform, and the Line described, were a mean Proportionall between the whole Length, and the half of the same.*

To the Proposition you object first, that it is all one whether the other motion be uniform or not, because the effect of each of their Motions, is but to carry the Body to the opposite side. But do you think that whatsoever be the Motions, the Body shall be carried by their concurrence alwayes to the same Point of the opposite side? If not, then the effect is not all one when a Motion is made by the concurrence of two Motions uniform and accelerated, and when it is made by the concurrence of two uniform or of two accelerated Motions.

Secondly, you say that these words, *and the Line described were a mean Proportionall between*

between the whole Length, and the half of the same, have no sense, or that you are deceived. True. For you are deceived; or rather you have not understanding enough distinctly to conceive variety of Motions though distinctly expressed. For when a Line is gone over with Motion uniformly accelerated, you cannot understand how a mean Proportionall can be taken between it and its half; or if you can, you cannot conceive that that mean can be gone over with uniform motion in the same Time that the whole Line was run over by motion uniformly accelerated. Yet these are things conceivable, and your want of understanding must be made my fault.

My Demonstration is this, In the Parallelogram $ABCD$, (Fig. 11.) Let the side AB be conceived to be moved uniformly till it lie in CD ; and let the Time of that Motion be AC , or BD . And in the same Time let it be conceived that AC is moved with uniform acceleration, till it lie in BD . To which you object, that then the acceleration last acquired must be far greater than that wherewith AB is moved uniformly; else it shall never come to the place you would have it in the same Time. What proof bring you for this? None here. Where then? No where that I remember. On the contrary I have proved (Art. 9. of this Chapter) that the Line described by the concurrence of those two Motions, namely, uniform from AB to CD , and uniformly accelerated from AC to BD , is the crooked Line of the Semiparabola AHD . And though I had not, yet it is well known that the same is demonstrated by Galileo. And seeing it is manifest that in what Proportion the Motion is accelerated in the Line AB , in the same Proportion the *Impetus* beginning from Rest in A is encreased in the same Times (which *Impetus* is designed all the way by the ordinate Lines of the Semiparabola) the greatest *Impetus* acquired must needs be the Base of the Semiparabola, namely BD , equal to AC which designs the whole Time. I cannot therefore imagine what should make you say without proof, that the greatest acquired *Impetus* is greater than that which is designed by the Base BD . Next you say, you see not to what end I divide AB in the middle at E . No wonder; for you have seen nothing all the way. Others would say it is necessary for the Demonstration; as also that the Point F is not to be taken arbitrarily; and likewise that the thirteenth Article (which you admit not for proof) is sufficiently demonstrated, and your objections to it answered. By the way you advise me, where I say *percursum eodem motu uniformi, cum Impetu ubique* &c. to blot out *cum*; because the *Impetus* is not a companion in the way, but the cause. Pardon me in that I cannot take your Learned Counsell; for the word *motu uniformi* is the Ablative of the Cause, and *Impetu* the Ablative of the Manner. But to come again to your objections, you say I make a greater space run over in the same Time by the slower Motion than by the swifter. How does that appear? because there is no doubt, but the swiftness is greater where the greatest *Impetus* is alwaies maintained, then where it is attained to in the same Time from Rest. True, But that is when they are considered asunder without concurrence, but not then when by the concurrence they debilitate one another, and describe a third Line different from both the Lines, which they would describe singly. In this place I compare their effects as contributing to the description of the Parabolicall Line AHD . What the effects of their severall Motions are, when they are considered asunder, is sufficiently shewn before in the first Article. You should first have gotten into your mindes the perfect and distinct Ideas of all the Motions mentioned in this Chapter, and then have ventured upon the censure of them, but not before. And then you would have seen that the Body moved from A , describeth not the Line AC , nor the Line AB , but a third, namely the Semiparabolicall Line AHD .

Again, where I say, *Wherefore if the whole AB be uniformly moved to CD , in the same Time wherein AC is moved uniformly to FG* ; you ask me *whether with the same Impetus or not*. How is it possible that in the same time two unequal Lengths should be passed over with the same *Impetus*? But, why (say you) do you not tell us with what *Impetus* AC comes to FG ? What need is there of that, when all men know that in uniform Motion and the same Time, *Impetus* is to *Impetus*, as Length to Length? Which to have expressed had not been pertinent to the Demonstration. That which follows in the Demonstration *rursus suppono*

pono quod latus AC, &c. to these words, *ut ostensum est, Art. 12.* You confute with saying you have proved that Article to be false. But you may see now, if you please, at the same place that I have proved your objections to be frivolous.

After this you run on without any Argument against the rest of the Demonstration, shewing nothing all the way, but that the variety and concurrence of Motions, the Speculations whereof you have not been used to, have made you giddy.

To the nineteenth Article you apply the same objection which you made to the eighteenth. Which having been answered, it appears that from the very beginning of your *Elenchus* to this place all your objections (except such as are made to three or four mistakes of small importance in setting down my mind,) are meer Paralogisms, and such as are less pardonable than any Paralogism in *Orontius*, both because the Subject as less difficult is more easily mastered, and because the same faults are more shamefully committed by a Reprehender than by any other man.

I had once added to these nineteen Articles a twentieth, which was this, *If from a Point in the Circumference there be drawn a Chord, and a Tangent equall to it, the Angle which they make shall be double to the aggregate of all the Angles made by the chords of all the equal Arches into which the Arch given can possibly be divided.* Which Proposition is true, and I did when I writ it think I might have use of it. But be it, or the Demonstration of it true or false, seeing it was not published by me, it is somewhat barbarous to charge me with the faults thereof. No Doctor of Humanity but would have thought it a poor and wretched malice, publicly to examine and censure papers of Geometry never published, by what means soever they came into his hands. I must confess that in these words, *in such kind of Progression Arithmetical* (that is, which begins with 0) *the sum of all the Numbers taken together, is equall to half the Number that is made by multiplying the greatest into the least,* there is a great error; for by this account these Numbers, 0, 1, 2, 3, 4, taken together should be equall to nothing. I should have said they are equall to that Number which is made by multiplying half the greatest into the Number of the Terms. There was therefore, if those words were mine (for truly I have no Copy of them, nor have had since the Book was Printed, and I have no great reason, as any man may see, to trust your Faith) a great error in the writing, but not an erroneous opinion in the writer. The Demonstration so corrected is true. And the Angles that have the Proportions of the Numbers 1, 2, 3, 4, are in the Table of your *Elenchus*, Fig. 12. the Angles GAD, HDE, IEF, KFB. And if the Divisions were infinite, so that the first were not to be reckoned but as a cypher, the Angle CAB would be double to them all together. This mistake of mine, and the finding that I had made no use of it in the whole book, was the cause why I thought fit to leave it quite out. But your Professions, could not forbear to take occasion thereby, to commend your zeal against Leviathan to your Doctorships of Divinity, by censuring it.

Of the Faults that Occure in Demon- stration.

To the same egregious Professors of the Ma-
thematically in the University of Oxford.

LESSON V.

AT the sixteenth Chapter, your first exception is to the Definition of Proportionall Proportions, which is this, *Four Proportions are then Proportionall, when the first is to the second, as the third to the fourth.* The Reader will hardly believe that your exception is in earnest. You say I mean not by Proportionality the Quantity of the Proportions. Yes I do, Therefore I say again, that *four Proportions are then Proportionall, when the Quantity of the first Proportion, is to the Quantity of the second Proportion, as the Quantity of the third Proportion, to the Quantity of the fourth Proportion.* Is not my meaning now plainly enough expressed? Or is it not the same Definition with the former? But what do I mean (you will say) by the Quantity of a Proportion? I mean the determined greatness of it, that is, for example, in these Numbers, the Quantity of the Proportion of two to three, is the same with the Quantity of the Proportion of four to six, or five to nine; and again, the Quantity of the Proportion of six to four, is the same with the Quantity of the Proportion of nine to six, or of three to two. But now what do you mean by the Quantity of a Proportion? You mean that two and three, are the Quantities of the Proportion of two to three (for so *Euclide* calls them) and that six and four are the Quantities of the Proportion of six to four, which is the same with the Proportion of three to two. And by this Rule, one and the same Proportion shall have an infinite Number of Quantities; and consequently the Quantity of a Proportion can never be determined. I call one Proportion double to another, when one is equal to twice the other, as the Proportion of four to one, is double to the Proportion of two to one. You call that Proportion double where one Number, Line, or Quantity absolute is double to the other; so that with you the Proportion of two to one is a double Proportion. It is easie to understand how the Number two is double to one, but to what I pray you, is double the Proportion of two to one, or of one to two? Is not every double Proportion double to some Proportion? See whether this Geometry of yours can be taken by any man of sound mind for sense. But 'tis known (you say) that in Proportions double is one thing, and duplicate another; So that it seems to you, that in talking of Proportion men are allowed to speak senselessly. 'Tis known, you say. To whom? It is indeed in use at this day to call double duplicate, and triple triplicate. And it is well enough; for they are words that signifie the same thing. But that they differ (in what subject soever) I never heard till now. I am sure that *Euclide* whom you have undertaken to expound, maketh no such difference. And even there where he putteth these Numbers, one, two, four, eight, &c. for Numbers in double Proportion (which is the last Proposition of the ninth Element) he meaneth not that one to

TWO,

two, or two to one, is a double Proportion, but that every Number in that Progression is double to the number next before it; and yet he does not call it *Analogy dupla*, but *Duplicata*. This distinction in Proportions between *dupla* and *duplicata*, proceeded long after from want of knowledge that the Proportion of one to two is double to the Proportion of one to four; and this from ignorance of the different nature of Proportions of Excess, and Proportions of Defect. And you that have nothing but by tradition saw not the absurdities that did hang thereon.

In the second Article I make $E K$, (Fig. 1.) the third part of $L K$, which you say is false; and consequently the Proposition undemonstrated. And thus you prove it false. Let $A C$ be 10 $G C$, or $G K$, to $G L$, as eight to one (for seeing the Point G is taken arbitrarily, we may place it where we will, &c.) and upon this placing of G arbitrarily, you prove well enough that $E K$ is not a third part of $L K$. But you did not then observe, that I make the Altitude $A G$, less than any Quantity given, and by consequence $E K$ to differ from a third part by a less difference than any Quantity that can be given. Therefore as yet the Demonstration proceedeth well enough. But perceiving your oversight, you thought fit (though before, you thought this confutation sufficient) to endeavour to confute it another way; but with much more evidence of ignorance. For when I come to say, the Proportion therefore between $A C$ and $G C$ is triple (in Arithmetical Proportion) to the Proportion between $G K$ and $G E$, &c. you say the Proportion of $A C$ to $G C$, is the Proportion of Identity, as also that of $G K$ to $G E$. But why? Does my construction make it so? Do no: I make $G C$ less than $A C$, though with less Difference than any Quantity that can be assigned? And then where I say therefore $E K$ is the third part of $L K$, you come in (by Parenthesis) with (or a fourth, or a fifth &c.) Upon what ground? Because you think it will pass for current, without proof, that a Point is nothing. Which if it do, Geometry also shall pass for nothing, as having no ground nor beginning but in nothing. But I have already in a former Lesson sufficiently shew'd you the consequence of that opinion. To which I may add, that it destroys the method of *Indivisibles*, invented by *Bonaventura*; and upon which, not well understood, you have grounded all your scurvy book of *Arithmetica Infinitorum*; where your *Indivisibles* have nothing to do, but as they are supposed to have Quantity, that is to say, to be *Divisibles*. You allow, it seems, your own nothings to be somethings, and yet will not allow my somethings to be considered as nothing. The rest of your objections having no other ground then this, that a Point is nothing, my whole Demonstration standeth firm; and so do the Demonstrations of all such Geometricians; Ancient and Modern, as have inferred any thing in the manner following, viz. If it be not greater nor less, then it is equal. But it is neither greater nor less. Therefore, &c. If it be greater, say by how much. By so much. It is not greater by so much. Therefore it is not greater. If it be less, say by how much, &c. Which being good Demonstrations are together with mine overthrown by the nothingness of your Point, or rather of your understanding; upon which you nevertheless have the vanity of advising me what to do, if I demonstrate the same again; meaning I should come to your false, impossible, and absurd Method of *Arithmetica Infinitorum*, worthy to be gilded, I do not mean with Gold.

And for your Question, why I set the Base of my Figure upwards, you may be sure it was not because I was afraid to say, that the Proportions of the ordinate Lines beginning at the Vertex were triplicate, or otherwise multiply of the Proportions of the intercepted parts of the Diameter. For I never doubted to call double duplicate, nor triple triplicate, &c. or if I had, I should have avoided it afterwards at the tenth Article of the same Chapter. But because when I went about to compare the Proportions of the ordinate Lines with those of their contiguous Diameters, the first thing I considered in them was in what manner the Base grew less and less till it vanished into a Point. And though the base had been placed below, it had not therefore required any change in the Demonstration. But I was the more apt to place the Base uppermost, because the Motion began at the Base, and ended at the Vertex. To proceed which way I pleased was in my own choice; and it is of grace that I give you any account of it at all.

To the third Article together with its Table, you say, it falls in the ruine of the second; and that the same is to be understood of the sixth, seventh, eighth and ninth. For confutation whereof I need to say no more, but that they all stand good by the confutation of your objections to the second.

To the fourth Article you say, the description of those curvilineall Figures is easie. True, to some men; and now that I have shewed you the way, 'tis easie enough for you also. For the way you propound is wholly transcribed out of the Figure of the second Article, which Article you had before rejected. For seeing the Lines HF , GE , AB , &c. are equall to the Lines CQ , CO , CD ; and the Lines QF , OE , DB equall to the Lines CH , CG , CA ; the Proportion of DB to OE , will be triple (that is, triplicate) to the Proportion of CO to GE ; and the Proportion of DB to QF , triple to the Proportion of CD to CQ ; and consequently, because the Complement $BDCFE B$ is made by the decrease of AC in triple Proportion to that of the decrease of CD , it will be (by the second Article) a third part of the Figure $ABEFCA$. So that it comes all to one pass, whether we take triple Proportion in decreasing to make the Complement, or triple Proportion in increasing to make the Figure; for the Proportion of HF to BA , is triple to the Proportion of CH to CA . Wherefore you have done no more but what you have seen first done, saying that from your construction you prove not the Figure to be triple to the Complement; perhaps because you have proved the contrary in your *Arithmetica Infinitorum*. But your way differs from mine, in that you call the Proportion subtriplicate, which I call triplicate; as if the divers naming of the same thing made it differ from itself. You might as well have said briefly, the Proposition is true, but ill proved, because I call the Proportion of one to two triple, or triplicate of that of one to eight; which you say is false, and hath infected the fourth, fifth, ninth, tenth, eleventh, thirteenth, fourteenth, fifteenth, sixteenth, seventeenth and nineteenth Articles of the sixteenth Chapter. But I say, and you know now, that it is true; and that all those Articles are Demonstrated.

Lastly you add, *Tu vero, in presente Articulo, &c. id est, you bid find as many mean Proportionals as one will, between two given Lines; as if that could not be done by the Geometry of Plaines, &c.* You might have left out *Tu vero* to seek an *Ego quidem*. But tell me, do you think you can find two mean Proportionals (which is less then as many as one will) by the Geometry of Plaines? We shall see anon how you go about it. I never said it was impossible, and if you look upon the places cited by you more attentively, you will find your self mistaken. But I say, the way to do it has not been yet found out, and therefore it may prove a solid Problem for any thing you know.

The fifth Article you reject, because it citeth the Corollarie of the twenty-eighth Article of the thirteenth Chapter, where there is never a word to that purpose. But there is in the twenty-sixth Article; which was my own fault, though you knew not but it might have been the Printers.

To the tenth you object for almost three leaves together, against these words of mine, *Because (in the sixth Figure) BE is to BE in triplicate Proportion of CD to FE , therefore inverting, FE is to CD in triplicate Proportion of BE to BC .* This you objected then. But now that I have taught you so much Geometry, as to know that of three Quantities, beginning at the least, if the third be to the first in triplicate Proportion, of the second to the first, also by conversion the first to the second shall be in triplicate Proportion of the first to the third; if it were to do again, you would not object it.

My eleventh Article you would allow for demonstrated, if my second had been Demonstrated, upon which it dependeth. Therefore seeing your objections to that Article are sufficiently answered, this Article also is to be allowed.

The twelfth also is allowed upon the same reason. What fallities you shall finde in such following Propositions as depend upon the same second Article, we shall then see when I come to the places where you object against them.

To the thirteenth Article you object, *That the same Demonstration may be as well applied to a Portion of any Conocides Parabolicall, Hyperbolicall, Ellipticall, or any other, as to the Portion of a Sphere.* By the truth of this let any man judge of your and my Geometry. Your Comparison of the Sphere and Conocides, so far holds good, as to prove that the Superficies of the Conocides is greater then the Superficies of the Cone described by the subtense of the Parabolicall, Hyperbolicall, or Ellipticall Line. But when I come to say that *The cause of the excess of the Superficies of the Portion of the Sphere above the Superficies of the Cone, consists in the Angle D A B, and the cause of the excess of the Circle made upon the Tangent A D above the Superficies of the same Cone, consists in the magnitude of the same Angle D A B,* how will you apply this to your Conocides? For suppose that the crooked Line A B (in the seventh Figure) were not an Arch of a Circle, do you think that the Angles which it maketh with the Subtense A B, at the Points A and B must needs be equal? Or if they be not, does the excess of the Superficies of the Circle upon A D above the Superficies of the Cone, or the excess of the Superficies of the Portion of the Conocides above the Superficies of the same Cone consist in the Angle D A B, or rather in the magnitude of the two unequal Angles D A B, and A B A? You should have drawn some other crooked Line, and made Tangents to it through A and B, and you would presently have seen your error. See how you can answer this; for if this Demonstration of mine stand firm, I may be bold to say, though the same be well Demonstrated by Archimedes, that this way of mine is more naturall, as proceeding immediately from the naturall efficient causes of the effect contained in the conclusion; and besides, more brief and more easie to be followed by the fancy of the Reader.

To the fourteenth Article you say that I commit a Circle in that I require in the fourth Article the finding of two mean Proportionals, and come not till now to shew how it is to be done. Nor now neither. But in the mean time you commit two mistakes in saying so. The place cited by you in the fourth Article is (in the Latine) Pag. 149. Lin. 9. (in the English) Pag. 188. Lin. 3. Let any Reader judge whether that be a requiring it, or a supposing it to be done; this is your first mistake. The second is, that in this place the Proposition it self, which is, *If these Deficient Figures could be described in a Parallelogram exquisitely, there might be found thereby between any two Lines given, as many mean Proportionals as one would,* is a Theoreme, upon supposition of these crooked Lines exquisitely drawn; but you take it for a Probleme.

And proceeding in that error, you undertake the invention of two mean Proportionals, using therein my first Figure, which is of the same construction with the eighth that belongeth to this fourteenth Article. Your construction is, *Let there be taken in the Diameter C A (Figure 1.) the two given Lines, or two others Proportionall to them, as C H, C G, and their ordinate Lines H F, G E (which by construction are in subtriplicate Proportion of the intercepted Diameters.) These Lines will shew the Proportions which those four Proportionals are to have.* But how will you find the Length of H F or G E, the ordinate Lines? Will you not do it by so drawing the crooked Line C F E as it may pass through both the Points F and E? You may make it pass through one of them, but to make it pass through the other, you must finde two mean Proportionals between G K and G L, or between H I and H P; Which you cannot do, unless the crooked Line be exactly drawn; which it cannot be by the Geometry of Plaines. Go shew this Demonstration of yours to Orontius, and see what he will say to it.

I am now come to an end of your objections to the seventeenth Chapter, where you have an Epiphonema not to be passed over in silence. But because you pretend to the Demonstration of some of these Propositions by another Method in your *Arithmetica Infinitorum*, I shall first try whether you be able to defend those Demonstrations as well as I have done these of mine by the Method of Motion.

The first Proposition of your *Arithmetica Infinitorum* is this Lemma. *In a Series (or Row) of Quantities Arithmetically Proportionall, beginning at a Point or Cypher as 0, 1, 2, 3, 4, &c. to finde the Proportion of the Aggregate of them all, to the Aggregate* of

of so many times the greatest, as there are Terms. This is to be done by multiplying the greatest into half the Number of the Terms. The Demonstration is easy. But how do you demonstrate the same? The most simple way (say you) of finding this and some other Problems, is to do the thing itself a little way, and to observe and compare the appearing Proportions, and then by Induction, to conclude it universally. Egregious Logicians and Geometricians, that think an Induction without a Numeration of all the particulars sufficient to infer a Conclusion universal, and fit to be received for a Geometrical Demonstration! But why do you limit it to the natural consequence of the Numbers, 0, 1, 2, 3, 4, &c? Is it not also true in these Numbers, 0, 2, 4, 6, &c. or in these, 0, 7, 14, 21, &c? Or in any Numbers where the Difference of nothing and the first Number is equal to the difference between the first and second, and between the second and third &c? Again, are not these Quantities 1, 3, 5, 7, &c. in continual Proportion Arithmetical? And if you put before them a Cypher thus, 0, 1, 3, 5, 7, do you think that the sum of them is equal to the half of five times seven? Therefore though your Lemma be true, and by me (Chap. 13. Art. 5.) demonstrated; yet you did not know why it is true; which also appears most evidently in the first Proposition of your Conique-sections. Where first you have this, *That a Parallelogram whose Altitude is infinitely little, that is to say, none, is scarce any thing else but a Line.* Is this the Language of Geometry? How do you determine this word scarce? The least Altitude, is Somewhat or Nothing. If Somewhat, then the first character of your Arithmetical Progression must not be a cypher; and consequently the first eighteen Propositions of this your *Arithmetica Infinitorum* are all naught. If Nothing, then your whole figure is without Altitude, and consequently your Understanding naught. Again, in the same Proposition, you say thus, *We will sometimes call those Parallelograms rather by the name of Lines than of Parallelograms, at least, when there is no consideration of a determinate Altitude; But where there is a consideration of a determinate Altitude (which will happen sometimes) there that little Altitude shall be so far considered, as that being infinitely multiplied it may be equal to the Altitude of the whole Figure.* See here in what a confusion you are when you resist the truth. When you consider no determinate Altitude (that is, no Quantity of Altitude) then you say your Parallelogram shall be called a Line. But when the Altitude is determined (that is, when it is Quantity) then you will call it a Parallelogram. Is not this the very same doctrine which you so much wonder at and reprehend in me, in your objections to my eighth Chapter, and your word considered used as I used it? 'Tis very ugly in one that so bitterly reprehendeth a doctrine in another, to be driven upon the same himself by the force of truth when he thinks not on't. Again, seeing you admit in any case, those infinitely little altitudes to be quantity, what need you this limitation of yours, so far forth as that by multiplication they may be made equal to the Altitude of the whole figure? May not the half, the third, the fourth, or the fifth part, &c. be made equal to the whole by multiplication? Why could you not have said plainly, so far forth as that every one of those infinitely little Altitudes be not only something but an aliquot part of the whole? So you will have an infinitely little Altitude, that is to say a Point, to be both nothing and something and an aliquot part. And all this proceeds from not understanding the ground of your Profession.

Well, the Lemma is true. Let us see the Theorems you draw from it. The first is (Pag. 3.) that a Triangle to a Parallelogram of equal Base and Altitude is as one to two. The conclusion is true, but how know you that? Because (say you) the Triangle consists as it were (as it were, is no Phrase of a Geometrician) of an Infinite Number of straight Parallel Lines. Does it so? Then by your own doctrine, which is, that Lines have no breadth, the Altitude of your Triangle consisteth of an infinite Number of no Altitudes, that is of an infinite Number of Nothings, and consequently the Area of your Triangle has no Quantity. If you say that by the Parallels you mean infinitely little Parallelograms, you are never the better; for if infinitely little, either they are nothing, or if somewhat, yet seeing that no two sides of a Triangle are Parallel, those Parallels cannot be Parallelograms. I see they may be counted for Parallelo-

Parallelograms by not considering the Quantity of their Altitudes in the Demonstration. But you are barred of that Plea, by your spitefull arguing against it in your *Elenchus*. Therefore this third Proposition, and with it the fourth is undemonstrated.

Your fifth Proposition is, *The Spirall Line is equal to half the Circle of the first Revolution.* But what Spirall Line? We shall understand that by your construction, which is this, *The straight Line M A*, (in your Figure which I have placed at the end of the fifth Lesson) *turned round (the Point M remaining unmoved) is supposed to describe with its Point A the Circle A O A*, whilst some Point (in the same *M A* whilst it goes about) is supposed to be moved uniformly from *M* to *A* describing the Spirall Line. This therefore is the Spirall Line of Archimedes; and your Proposition affirms it to be equal to the half of the Circle *A O A*; which you perceived not long after to be false. But thinking it had been true, you go about to prove it, by inscribing in the Circle an infinite multitude of equal Angles, and consequently an infinite Number of Sectors, whose Arches will therefore be in Arithmetical Proportion; Which is true. And the Aggregate of those Arches equal to half the Circumference *A O A*; Which is true also. And thence you conclude that the Spirall Line is equal to half the Circumference of the Circle *A O A*; Which is false. For the Aggregate of that infinite Number of infinitely little Arches, is not the Spirall Line made by your construction, seeing by your construction the Line you make is manifestly the Spiral of Archimedes; whereas no Number (though infinite) of Arches of Circles (how little soever) is any kind of Spirall at all; and though you call it a Spirall, that is but a patch to cover your fault, and deceiveth no man but your self. Besides, you saw not how absurd it was (for you that hold a Point to be absolutely nothing) to make an infinite Number of equal Angles (the Radius increasing as the Number of Angles increaseth) and then to say that the Arches of the Sectors whose Angles they are, are as 0, 1, 2, 3, 4, &c. For you make the first Angle 0, and all the rest equal to it; and so make 0, 0, 0, 0, &c. to be the same Progression with 0, 1, 2, 3, 4, &c. The influence of this absurdity reacheth to the end of the eighteenth Proposition. So many are therefore false, or nothing worth. And you needed not to wonder that the Doctrine contained in them was omitted by Archimedes, who never was so senseless as to think a Spirall Line was compounded of Arches of Circles.

Your nineteenth Proposition is this other Lemma, *In a Series (or a Row) of Quantities, beginning from a Point, or Cypher, and proceeding according to the order of the square Numbers, as 0, 1, 4, 9, 16, &c. to find what Proportion the whole Series hath to so many times the greatest.* And you conclude the Proportion to be that of 1 to 3. Which is false, as you shall presently see. First, let the Series of Squares with the prefixed Cypher, and under every one of them, the greatest 4 be $\frac{0.1.4.9}{4.4.4}$. And you have for the sum of the Squares 5, and for thrice the greatest 12, the third part whereof is 4. But 5 is greater then 4, by 1, that is, by one twelfth of 12; which Quantity is somewhat, let it be called A. Again, let the Row of Squares be lengthened one term further, and the greatest set under every one of them as $\frac{0.1.4.9}{9.9.9.9}$. The sum of the Squares is 14, and the sum of four times the greatest is 36, whereof the third part is 12. But 14 is greater then 12 by two unities, that is, by two twelfths of 12, that is, by 2 A. The difference therefore between the sum of the Squares, and the sum of so many times the greatest Square is greater, when the cypher is followed by three Squares, then when by but two. Again, let the Row have five terms as in these Numbers $\frac{0.1.4.9.16}{16.16.16.16.16}$ with the greatest five times subscribed, and the sum of the Squares will be 30, the sum of all the greatest will be 80. The third part whereof is $26\frac{2}{3}$. But 30 is greater than

$26\frac{2}{3}$ by $3\frac{1}{3}$, that is, by three twelfths of twelve and $\frac{1}{3}$ of a twelfth, that is, by $3\frac{1}{3}$ A. Likewise in the Series continued to six places with the greatest six times subscribed, as

$0.1.4.9.16.25.$ the sum of the Squares is 55, and the sum of the greatest six times taken is 150, the third part where fis 50. But 55 is greater then 50 by 5, that is, by five twelfths of 12, that is, by 5 A. And so continually as the Row groweth longer, the excess also of the aggregate of the Squares above the third part of the aggregate of so many times the greatest Square, groweth greater. And consequently if the Number of the Squares were infinite, their sum would be so far from being equal to the third part of the aggregate of the greatest as often taken, as that it would be greater then it by a Quantity greater then any that can be given or named.

That which deceived you was partly this, that you think (as you do in your *Elenchus*) that these Fractions $\frac{1}{12} \frac{1}{18} \frac{1}{24} \frac{1}{30} \frac{1}{36}$ &c. are Proportions, as if $\frac{1}{12}$ were the Proportion of one to twelve, and consequently $\frac{2}{12}$ double the Proportion of one to twelve; which is as unintelligible as School-Divinity; and I assure you, far from the meaning of Mr. Oughtred in the sixth Chapter of his *Clavis Mathematicæ*, where he sayes that $4\frac{3}{7}$ is the Proportion of 31 to 7; for his meaning is, that the Proportion of $4\frac{3}{7}$ to one, is the Proportion of 31 to 7; whereas if he meant as you do, then $8\frac{6}{7}$ should be double the Proportion of 31 to 7. Partly also because you think (as in the end of the twentieth Proposition) that if the Proportion of the Numerators of these Fractions $\frac{1}{12} \frac{1}{18} \frac{1}{24} \frac{1}{30} \frac{1}{36}$ to their Denominators decrease eternally, they shall so vanish at last as to leave the Proportion of the sum of all the Squares to the sum of the greatest so often taken, (that is, an infinite Number of times) as one to three, or the sum of the greatest to the sum of the increasing Squares, as three to one; for which there is no more reason then for four to one, or five to one, or any other such Proportion. For if the Proportions come eternally nearer and nearer to the subtriple, they must needs also come nearer and nearer to subquadruple; and you may as well conclude thence that the upper Quantities shall be to the Lower Quantities as one to four, or as one to five, &c. as conclude they are as one to three. You can see without admonition, what effect this false ground of yours will produce in the whole structure of your *Arithmetica Infinitorum*; and how it makes all that you have said unto the end of your thirty-eighth Proposition, undemonstrated, and much of it false.

The thirty-ninth is this other Lemma, In a Series of Quantities beginning with a Point or Cypher, and proceeding according to the Series of the Cubique Numbers, as 0. 1. 8. 27. 64. &c. to finde the Proportion of the sum of the Cubes to the sum of the greatest Cube, so many times taken as there be Terms, And you conclude that they have the Proportion of 1 to 4; which is false.

Let the first Series be of three terms subscribed with the greatest $\frac{0.1.8}{8.8.8}$; the sum of the Cubes is nine; the sum of all the greatest is 24; a quarter whereof is 6. But 9 is greater then 6 by three unities. An unity is something. Let it be therefore A. Therefore the Row of Cubes is greater then a quarter of three times eight, by three A. Again, let the Series have four terms, as $\frac{0.1.8.27}{27.27.27.27}$; the sum of the Cubes is 36; a quarter of the sum

sum of all the greatest is twenty-seven. But thirty-five is greater than twenty-seven by nine, that is, by 3 A. The excess therefore of the sum of the Cubes above the fourth part of the sum of all the greatest, is increased by the increase of the Number of terms. Again, let the terms be five as $1^3, 2^3, 3^3, 4^3, 5^3$ the sum of the Cubes is one hundred; the sum of all the greatest three hundred and twenty 3; a quarter whereof is eighty. But one hundred is greater than eighty by twenty, that is, by 2 A. So you see that this Lemma also is false. And yet there is grounded upon it all that which you have of comparing *Parabolas* and *Paraboloides* with the *Parallelograms* wherein they are accommodated. And therefore though it be true, that the *Parabola* is $\frac{2}{3}$, and the Cubicall *Paraboloides* $\frac{3}{4}$ of their *Parallelograms* respectively,

yet it is more then you were certain of when you referred (me for the learning of Geometry) to this Book of yours. Besides, any man may perceive that without these two Lemmas (which are mingled with all your compounded Series with their excesses) there is nothing demonstrated to the end of your Book. Which to prosecute particularly, were but a vain expence of time. Truly were it not that I must defend my reputation, I should not have shewed the world how little there is of sound Doctrine in any of your Books. For when I think how dejected you will be for the future; and how the grief of so much time irrecoverably lost, together with the confidence of taking so great a stipend, for mis-teaching the young men of the University; & the consideration of how much your friends will be ashamed of you, will accompany you for the rest of your life, I have more compassion for you then you have deserved. Your Treatise of the *Angle of Contact*, I have before confuted in a very few leaves. And for that of your *Conique Sections*, it is so covered over with the scab of Symbols that I had not the patience to examine whether it be well or ill demonstrated.

Yet I observed thus much, that you find a *Tangent* to a Point given in the Section, by a *Diameter* given; and in the next Chapter after, you teach the finding of a *Diameter*, which is not artificially done.

I observe also, that you call the *Parameter* an Imaginary Line, as if the place thereof were left determined then the *Diameter* itself; and then you take a mean Proportionall between the intercepted *Diameter*, and its contiguous ordinate Line to find it. And tis true, you find it: But the *Parameter* has a determined Quantity to be found without taking a mean Proportionall. For the *Diameter* and half the Section being given, draw a *Tangent* through the Vertex, and dividing the Angle in the midst which is made by the *Diameter* and *Tangent*, the Line that so divideth the Angle, will cut the trooked Line from the intersection draw a Line (if it be a *Parabola*) Parallel to the *Diameter*, and that Line shall cut off in the *Tangent* from the Vertex the *Parameter* sought. But if the Section be an *Ellipsis*, or an *Hyperbole*, you may use the same Method, saving that the Line drawn from the Intersection must not be Parallel, but must pass through the end of the transverse *Diameter*, and then also it shall cut off a part of the *Tangent*, which measured from the Vertex is the *Parameter*. So that there is no more reason to call the *Parameter* an Imaginary Line then the *Diameter*.

Lastly, I observe that in all this your new Method of *Coniques* you shew not how to find the *Burning Points*, which writers call the *Foci* and *Umbilici* of the Section, which are of all other things belonging to the *Coniques* most usefull in Philosophy. Why therefore were they not as worthy of your pains as the rest, for the rest also have already been demonstrated by others? You know the *Focus* of the *Parabola* is in the *Axis* distant from the Vertex a quarter of the *Parameter*. Know also that the *Focus* of an *Hyperbole*, is in the *Axis*, distant from the Vertex, as much as the *Hypotenusal* of a rectangled Triangle, whose one side is half the transverse *Axis*, the other side half the mean Proportionall between the whole transverse *Axis* and the *Parameter*, is greater then half the transverse *Axis*.

The cause why you have performed nothing in any of your Books (saving that in your *Elements* you have spied a few negligences of mine, which I need not be ashamed of) is this, that

you understand not what is Quantity, Line, Superficies, Solid, and Proportion; without which you cannot have the Science of any one Proposition in Geometry. From this one and first Definition of Euclid, a Point is thus defined: there is no part, understood by Sensus Empiricus, as you understand it, that is to say, mis-understood, Sensus Empiricus hath utterly destroyed most of the rest, and Demonstrated, that in Geometry there is no Science; and by that means you have betrayed the most evident of the Sciences to the Sceptiques. But as I understand it for that whereof no part is reckoned, his Arguments have no force at all, and Geometry is redeemed. If a Line have no Latitude, how shall a Cylinder rowling on a Plain, which it toucheth not but in a Line, describe a Superficies? How can you affirm that any of those things can be without Quantity, whereof the one may be greater or less then the other? But in the common Contact of divers Circles the externall Circle maketh with the common Tangent a less Angle of Contact then the internall. Why then is it not Quantity? An Angle is made by the concurrence of two Lines from severall Regions, concurring (by their generation) in one and the same Point. How then can you say the Angle of Contact is no Angle? One measure cannot be applicable at once to the Angle of Contact, and Angle of Conversion. How then can you infer, if they be both Angles, that they must be Homogeneous? Proportion is the Relation of two Quantities. How then can a Quotient or Fraction, which is Quantity absolute, be a Proportion? But to come at last to your Epiphonema, wherein, though I have perfectly demonstrated all those Propositions concerning the Proportion of Parabolas to their Parallelograms, and you have demonstrated none of them (as you cannot now but plainly see) but committed most gross Paralogisms, How could you be so transported with pride, as insolently to compare the setting of them forth as mine, to the Act of him that steals a horse, and comes to the gallows for it. You have read, I think, of the gallows set up by Haman. Remember therefore also who was hanged upon it.

After your dejection I shall comfort you a little, a very little, with this, that whereas this 18 Chapter containeth two Problems, one, the finding of a straight Line equal to the crooked Line of a Semiparabola, The other, the finding of straight Lines equal to the crooked Lines of the Parabolas, in the table of the third Article of the 17 Chapter. You have truly demonstrated that they are both false; and another hath also Demonstrated the same another way. Nevertheless the fault was not in my method, but in a mistake of one Line for another, and such as was not hard to correct; and is now so corrected in the English as you shall not be able (if you can sufficiently imagine Motions), to reprehend. The fault was this, That in the Triangles which have the same Base and Altitude with the Parabola, and Parabolaster, I take for designation of the mean uniform Impetus, a mean Proportionall (in the first Figure) between the whole Diameter and its half, and (in the second Figure) a mean Proportionall between the whole Diameter and its third part; which was manifestly false, and contrary to what I had shewn in the 16 Chapter. Whereas I ought to have taken the Last of the Base, as now I have done, and thereby exhibited the Straight Lines equal to those crooked Lines, as I undertook to do. Which error therefore proceeded not from want of skill, but from want of care; and what I promised (as bold as you say the promise was,) I have now performed.

The rest of your exceptions to this Chapter are to these words in the end, there be some that say, that though there be equality between a straight and crooked Line, yet now, (they say) after the fall of Adam, it cannot be found without the especial help of divine grace. And you say you think there be none that say so. I am not bound to tell you who they are. Nevertheless, that other men may see the Spirit of an ambitious part of the Clergy, I will tell you where I read it. It is in the Prolegomena of Lalouera (a Jesuite) to his Quadrature of the Circle, pag. 13 & 14, in these words, *Quamvis circuli, et polygoni sit quædam possibilis, et tamam etiam sit meos hunc, hoc est, post Adæ lapsum homo ejus scientiam absq. speciali divina gratia auxilio, possit comparare, jure merito inquirunt Theologi, pronunciantes, hanc veritatem tanta esse caligine involutam ut illam videre nemo possit, nisi Ignorantia ex pri-*

qui parentis pronuntiatione propagat, et cunctas in debitis divinis institutionibus dissipat, quod verissimum esse sentio. Wherein I observed that he (supposing he had found that Quadrature) would have us believe it was not by the ordinary and Naturall help of God (whereby one man reasoneth, judgeth, and remembereth better then another) but by a Special (which must be a Super-entall) help of God, that he hath given to him of the order of Jesus above others that have attempted the same in vain. Intimating thereby, as handsomely as he could, a Speciall love of God towards the Jesuites. But you taking no notice of the word *Speciall*, would have men think I held, that humane Sciences might be acquired without any help of God. And there-upon proceed in a great deal of ill language to the end of your objections to this Chapter. But I shall take notice of your Manners for altogether in my next Lesson.

As the nineteenth Chapter you see not (you say) the Method. Like enough. In this Chapter I consider not the Cause of Reflection, which consisteth in the resistance of Bodies naturall; but I consider the consequences, arising from the supposition of the equality of the Angle of Reflection, to that of Incidence; leaving the causes both of Reflection, and of Refraction to be handled together in the 24 Chapter. Which Method (think what you will) I still think best.

Secondly, you say I define not here, but many Chapters after, what an Angle of Incidence, and what an Angle of Reflection is. Had you not been more hasty then diligent Readers, you had found that those Definitions of the Angle of Incidence, and of Reflection were here set down in the first Article, and not deferred to the 24. Let not therefore your own oversight be any more brought in for an objection.

Thirdly, you say there is no great difficulty in the business of this Chapter. It may be so, now 'tis down; but before it was done, I doubt not but you that are a Professor would have done the same, as well as you have done that of the Angle of Contact, or the business of your *Arithmetica Infinitorum*. But what a novice in Geometry would have done I cannot tell.

To the third, fourth, and fifth Article, you object a want of Determination; and shew it by instance, as to the third Article. But what those Determinations should be, you determine not, because you could not. The words in the third Article, are first these, *If there fall two straight Lines Parallel, &c.* which is too generall. It should be, *if there fall the same way two straight Lines Parallel, &c.* Next these, *their Reflected Lines produced inwards shall make an Angle, &c.* This also is too generall. I should have said *their Reflected Lines produced inwards, if they meet within, shall make an Angle, &c.* Which done, both this Article and the 4 and 5 are fully demonstrated. And without it, an intelligent Reader had been satisfied, supplying the want himself by the construction.

To the eight, you object onely the two great Length, and labour of it, because you can do it a shorter way. Perhaps so now, as being easie to shorten many of the Demonstrations both of Euclide, and other the best Geometricians that are or have been. And this is all you had to say to my 19 Chapter. Before I proceed, I must put you in mind that these words of yours, *Adducio malleum, ut occidat muscam*, are not good Latine, *Malleum affers, Malleum adhibes, Malleo uteris*, are good. When you speak of bringing Bodies animate, *Ducere* and *Adducere* are good, for there *to bring*, is *to guide or lead*, And of Bodies inanimate *Adducere* is good for *Attrahere*, which is to draw to. But when you bring a hammer, will you say *Adduco Malleum, I lead a hammer*? A man may lead another man, and a ninny may be said to lead another ninny, but not a hammer. Nevertheless, I should not have thought fit to reprehend this fault upon this occasion in an English man, nor to take notice of it, but that I finde you in some places nibbling (but causelessly) at my Latine.

Concerning the twentieth Chapter, before I answer to the objections against the Propositions themselves, I must answer to the exception you first take to these words of mine, *Quæ de dimensione Circuli & Angulorum pronuntiata sunt tanquam exactè inventa, accipiat Lector tanquam dicta Problematica*. To which you say thus, *we are wont in Geometry to call some Pro-*

positions Theorems, about Problems, &c. of which I have said that wherein some assertion is propounded to be proved, a Problem that wherein something is commanded to be done. Do you mean to be done, and not proved? By your favour, a Problem in all ancient writers signifies no more but a Proposition uttered to the end to have it by them, to whom it is uttered, examined whether it be true or not true, feasible or not feasible; and differs not amongst Geometricians from a Theorem; but in the manner of Propounding. For this Proposition, To make an equilateral Triangle, so propounded they call a Problem. But if propounded thus, If upon the ends of a straight Line given be described two Circles, whose Radius is the same straight Line, and there be drawn from the intersection of the Circles to their two Centers, two straight Lines, there will be made an equilateral Triangle, then they call it a Theorem; and yet the Proposition is the same. Therefore these words *decipiat Lector tanquam dicta Problematicey*, signifie plainly this; that I would have the Reader, take for propounded to him to examine, whether from my construction the Quadrature of the Circle can be truly inferred or not; and this is not to bid him (as you interpret it) to square the Circle. And if you believe that *Problematicey* signifies probably, you have been very negligent in observing the sense of the ancient Greek Philosphers in the word *Problema*. Therefore your *Solemus in Geometria, &c.* is nothing to the purpose; nor had it been though you had spoken more properly, and said *Solent*, leaving out yourselves.

My first Article hath this Title, from a False Supposition, a false Quadrature of the Circle. Seeing therefore you were resolved to shew where I erred, you should have proved either that the Supposition was true, and the Conclusion falsely inferred, or contrarily that though the Supposition be false, yet the Conclusion is true; for else you object nothing to my Geometry, but only to my Judgement, in thinking fit to publish it; which nevertheless you cannot justly do, seeing it was likely to give occasion to ingenious men (they practise of it being so accurate to sense) to inquire wherein the Fallacy did consist. And for the Probleme as it was first printed, but never published, and consequently ought to have passed for a private paper stoln out of my study, your publique objecting against it, (in the opinion of all men that have conversed so much with honest company as to know what belongs to civill conversation,) was sufficiently barbarous in Divines. And seeing you knew I had rejected that Proposition, it was but a poor Ambition to take wing as you thought to do, like Beesles from my egestions. But let that be as it will, you will think strange now I should resume, and make good (at least against your objection) that very same Proposition. So much of the Figure as is needfull you will finde noted with the same letters, and placed at the end of this 5 Lesson. Wherein let BI be an Arch not greater then the Radius of the Circle, and divided into four equal parts, in L, N, O . Draw SN , the Sine of the Arch BN , and produce it to T , so as ST be double to SN , that is, equal to the Chord BI . Draw likewise aL , the Sine of the Arch BL , and produce it to c , so as ac be quadruple to aL , that is, equal to the two Chords BN, NI . Upon the Center N with the Radius NI , draw the Arch Id , cutting BU the Tangent in d . Then will BN produced cut the Arch Id , in the midst at e . In the Line BS produced take Sb equal to BS ; then draw and produce bN , and it will fall on the Point d . And Bd, ST , will be equal; and dT joyned and produced will fall upon c , the midst of the Arch Id . Joyn IT , and produce it to the Tangent BU in U . I say, that the straight Line ITU shall pass through c . For seeing BS, Sb , are equal, and the Angle at S a right Angle, the straight Lines BN , and bN , are also equal, and the Triangles BNb, dNo like and equal; and the Lines dT, Td equal. Draw oi Parallel to dU , cutting IU in i ; and the Triangles dTv, oTi will also be like and equal. Produce ST to the Arch $d o I$ in e , and produce it further to f , so that the Line ef be equal to Tc ; and then Sf will be equal to ac . Therefore fec joyned will be Parallel to BS . In cf produced take fg equal to cf ; and draw gm Parallel to dU , cutting IU in m , and $d o$ in n ; and let the intersection of the two Lines ac and $d o$ be in r ; which being done, the Triangles $m n T, r c T$ will be like and equal. Therefore $m n$ and rc are equal; and consequently the straight Line $I m T U$ shall pass through c . Dividing therefore ac in the midst at r , and SN

in the midst at q , and joyning N , L , the Lines L , N , and T produced with all meet in one and the same Point of B Sprouced; suppose at q . Therefore the Point q being given by the two known Points T and L , the Lines drawn from q through equall parts of the Sine of the Arch BI , for example through the Points P , Q , R , of the Sine MI , shall cut off equall Arches, as BL , LN , NO , OL . And this is enough to make good that Probleme, as to your objection.

The straight Line therefore BU for any thing you have said is proved equall to the Arch BI , and the division of any Angle given into any proportion given, the Quadrature of any Sector, and the Construction of any equilaterall Polygon is also given. And though in this also I should have etred, yet it cannot be denied but that I have used a more natural, a more Geometrical, and a more peispicuous method in the search of this so difficult a Probleme, then you have done in your *Arithmetica Infinitorum*. For though it be true that the aggregate of all the mean Proportionals between the Radius together with an infinitely little part of the same, and the Radius wanting an infinitely little part of the same; and again, between the Radius, together with two infinitely little parts, and the Radius wanting two infinitely little parts, and so on eternally will be equall to the Quadrant (a thing which every mean Geometrician knew before) yet it was absurd to think those Means could be calculated in Numbers by *Interpoling* of a Symbole; especially when you make that Symbole to stand for a Number neither true nor surd; as if there were a number that could neither be uttered in words, nor not be uttered in words. For what else is surd, but that which cannot be spoken?

To the fifth Article (though your discourse be long) you object but two things. One is, that *Whereas the Spirall of Archimedes is made of two Motions, one straight, the other circular, both uniform, I taking the Motion compounded of them both for one of those that are compounded, conclude falsely, that the generation of the Spirall is like to the generation of the Parabola*. What heed you use to take in your rep.ensions, appears most manifestly in this objection. For I say in that demonstration of mine that *the velocity of the point A in describing the Spirall, encreaseth continually in Proportion to the Times*. For seeing it goes on uniformly in the Semidiameter, it is impossible it should not pass into greater and greater Circles proportionally to the Times; and consequently it must have a swifter and swifter Motion circular, to be compounded with the uniform Motion in every Point of the Radius as it turneth about. This objection therefore is nothing but an effect of a Will (without cause) to contradict.

The other objection is that *Granting all to be true hitherto, yet because it depends upon the finding of a straight Line equall to a Parabolicall Line in the 18 Chapter where I was deceived, I am also deceived here*. True. But because in the 18 Chapter of this English Edition I have found a straight Line equall to a Parabolicall Line, I have also found a straight Line equall to the Spirall Line of Archimedes. I must here put you in minde that by these words in your objections to the fifth Article at your Number 2, *Quatenus verum est, &c. we have demonstrated Prop. 10, 11, 13. Arithmet. Infin.* you make it appear that you thought your Spirall (made of Arches or Circles) was the true Spirall of Archimedes; which is fully as absurd as the Quadrature of Joseph Scaliger, whose Geometry you so much despise.

To the sixth Article which is a Digression concerning the Analytiques of Geometricians, you deny that *the Efficient cause of the Construction ought to be contained in the Demonstration*. As if any Probleme could be known to be truly done, otherwise then by knowing first how, that is to say, by what Efficient Cause, and in what manner it is to be done. Whatsoever is done without that knowledge, cannot be demonstrated to be done; as you see in your computation of the Parabola, and Parabolocides, in your *Arithmetica Infinitorum*.

And whereas I said that *The ends of all straight Lines drawn from a straight Line, and passing through one and the same Point, if their parts be Proportionall, shall be in a straight Line*; is true and accurate; as also *If they begin in the Circumference of a Circle, they shall also*

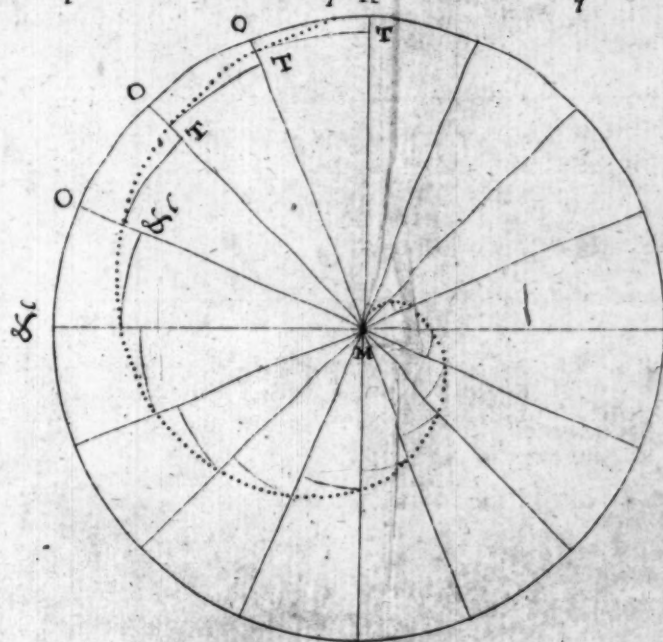
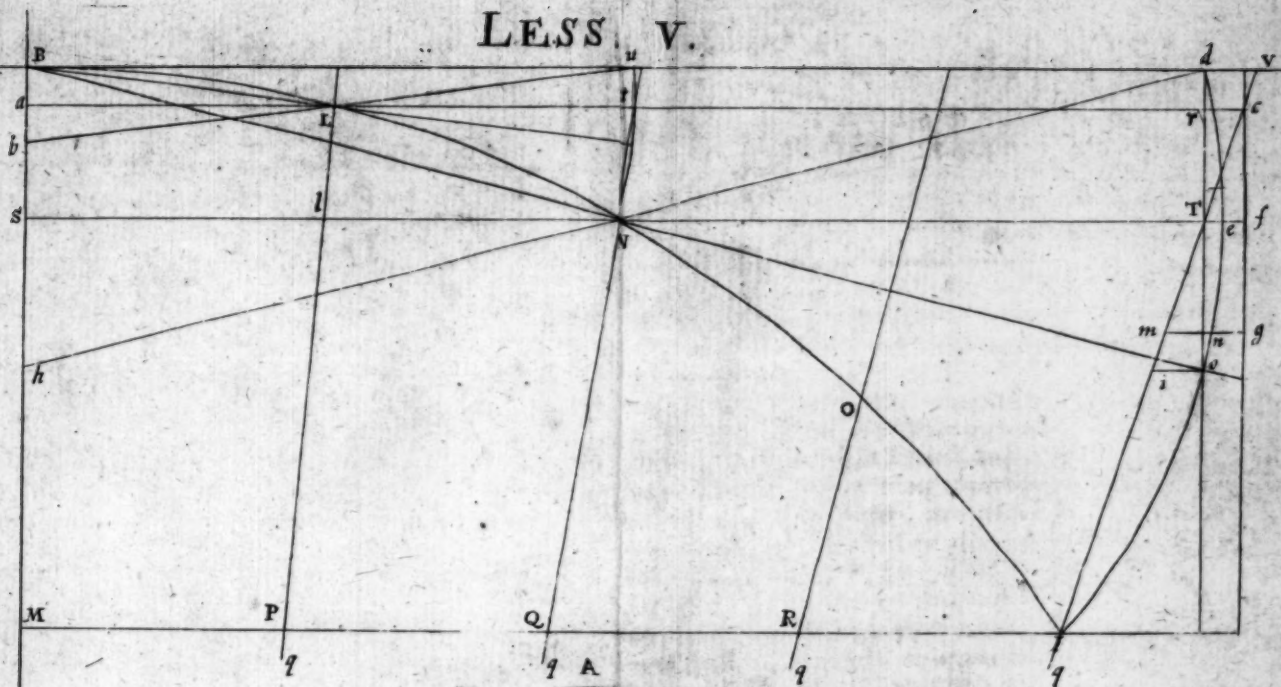
also be in the Circumference of another Circle. And so is this, If the Proportion be duplicate, they shall be in a Parabola. All this I say is true and accurately spoken. But this was no place for the demonstration of it. Others have done it. And I perceive by that you put in by Parenthesis (*intelligi credo inter duas Peripherias concentricas*) that you understand not what I mean.

Hitherto reach your objections to my Geometry. For the rest of your Book, it containeth nothing but a collection of lies, wherewith you do what you can, to extenuate as vulgar, and disgrace as false, that which followeth, and to which you have made no speciall objection.

I shall therefore only add in this place concerning your *Analytica per Potestates*, that it is no Art. For the Rule, both in Mr. Oughtred, and in *Der Carter* is this, *When a Probleme or Question is propounded, suppose the thing required done, and then using a fit ratiocination, put A or some other vowel for the magnitude sought.* How is a man the better for this Rule without another rule; *How to know when the ratiocination is fit?* There may therefore be in this kind of Analysis more or less naturall prudence, according as the Analyst is more or less wise, or as one man in chusing of the unknown Quantity with which he will begin, or in chusing the way of the consequences which he will draw from the *Hypothesis*, may have better luck then another. But this is nothing to Art. A man may sometimes spend a whole day in deriving of consequences in vain, and perhaps another time solve the same Probleme in a few minutes.

I shall also add, that Symboles though they shorten the writings, yet they do not make the Reader understand it sooner then if it were written in words. For the conception of the Lines and Figures (without which a man learneth nothing) must proceed from words, either spoken or thought upon. So that there is a double labour of the mind, one to reduce your Symboles to words (which are also Symboles) another to attend to the Ideas which they signifie. Besides, if you but consider how none of the Antients ever used any of them in their published demonstrations of Geometry, nor in their Books of Arithmetique, more then for the *Rootes* and *Potestates* themselves; and how bad success you have had your self in the unskilfull using of them, you will not, I think, for the future be so much in love with them as to demonstrate by them that first part you promise of your *Opera Mathematica*. In which if you make not amends for that which you have already published, you will much disgrace those Mathematicians you address your Epistles to, or otherwise have commended; as also the Universities (as to this kinde of Learning) in the sight of learned men beyond Sea. And thus having examined your panier of Mathematicques, and finding in it no knowledge neither of Quantity, nor of measure, nor of Proportion, nor of Time, nor of Motion, nor of any thing, but only of certain Characters, as if a Hen had been scraping there; I take out my hand again, to put it in to your other panier of Theology, and good Manners. In the mean time I will trust the objections made by you the Astronemer (wherein there is neither close reasoning, nor good stile, nor sharpness of wit, to impose upon any man) to the discretion of all sorts of Readers.

LESS. V.



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To the same egregious Professors of the Mathematicks in the University of Oxford.

LESSON VI.

HAVING in the precedent Lessons maintained the Truth of my Geometry, and sufficiently made appear, that your objections against it are but so many errors of your own, proceeding from misunderstanding of the Propositions you have read in *Euclide*, and other Masters of Geometry; I leave it to your consideration to whom belong (according to your own sentence) the unhandsome attributes you so often give me upon supposition, that you your selves are in the right, and I mistaken; and come now to purge my self of those greater accusations which concern my Manners. It cannot be expected there should be much Science of any kinde in a man that wanteth Judgement; nor Judgement in a man that knoweth not the Manners due to a publique disputation in writing; wherein the scope of either party ought to be no other then the examination and manifestation of the truth. For whatsoever is added of contumely, either directly, or scematically, is want of Charity, and uncivil; unless it be done by way of Reddition from him that is first provoked to it. I say unless it be by way of Reddition; for so was the Judgement given by the Emperor *Vespasian* in a quarrell between a Senato and a Knight of *Rome* which had given him ill language. For when the Knight had proved, that the first ill language proceeded from the Senator, the Emperor acquitted him in these words *Maledicti senatoribus non oportere; remaledicere, sat est tibi esse*. Nevertheless, now a dayes uncivil words are commonly and bitterly used by all that write in matter of Controversie, especially in Divinity, excepting now and then such writers as have been more then ordinarily well bred, and have observed, how hainous, and hazardous a thing such contumely is amongst some sorts of men, whether that which is said in disgrace be true or false. For evill words by all men of understanding are taken for a defiance; and a challenge to open war. But that you should have observed so much, who are yet in your mothers belly, was not a thing to be much expected.

The faults in Manners you lay to my charge, are these, 1. *Self conceit*. 2. *That I will be very angry with all men that do not presently submit to my Dictates*. 3. *That I had my Doctrine concerning Pison, out of papers which I had in my hands of Mr. Warners*. 4. *That I have injured the Universities*. 5. *That I am an Enemy to Religion*. These are great faults; but such as I cannot yet confess. And therefore I must as well as I can, seek out the grounds upon which you build your Accusations. Which grounds (seeing you are not acquainted with my conversation) must be either in my published writings, or reported to you by honest men, and without suspicion of interest in reporting it. As for my self-conceit and ostentation, you shall finde no such matter in my writings. That which you alleadge from thence is first that in the Epistle Dedicatory I say of my Book *de Corpore*, *Though it be little, yet it is full; and if good may go for great, great enough*. When a man presenting a gift great or small to his

bettors,

betters, adorneth it the best he can to make it the more acceptable; he that thinks this to be *Ostentation*, and *self-conceit*, is little versed in the common actions of humane life. And in the same Epistle where I say of *Civill Philosophy*, it is no *antienter* than my *Book de Cive*, these words are added, *I say it provoked, and that my detractors may see they lose their labour*. But that which is truly said, and upon provocation, is not *boasting*, but *defence*. A short sum of that Book of mine, now publicly in French, done by a Gentleman I never saw, carrieth the Title of *Ethiques demonstrated*. The Book it self translated into French hath not onely a great testimony from the Translator *Sorberius*, but also from *Gassendus*, and *Mersennus*, who being both of the Roman Religion had no cause to praise it; or the Divines of England have no cause to finde fault with it. Besides, you know that the Doctrine therein contained is generally received by all but those of the Clergy, who think their interest concerned in being made subordinate to the Civil Powe; whose testimonies therefore are invalide. Why therefore if I commend it also against them that dispraise it publicly, do you call it *boasting*? You have heard (you say) that I had promised the *Quadrature of the Circle*, &c. You heard then that which was not true. I have been asked sometimes, why such as saw the Figure before me, what I was doing, and I was not afraid to say I was seeking for the solution of that Problem; but not that I had done it. And afterwards being asked of the success, I have said, I thought it done. This is not *boasting*; and yet it was enough, when told again, to make a fool believe 'twas *boasting*. But you the Astronomer in the Epistle before your Philosophicall Essay, say you had a great expectation of my Philosophicall, and Mathematicall works, before they were published. It may be so. Is that my fault? can a man raise a great expectation of himself by *boasting*? If he could, neither of you would be long before you raised it of your selves; saving that what you have already published, has made it now too late. For I verily believe there was never seen worse reasoning then in that Philosophicall Essay; which any judicious Reader would believe proceeded from a *Prævaricator*, rather then from a man that believed himself; nor worse Principles, then those in your Books of Geometry. The expectation of that which should be written by me, was raised partly by the *Cogitata Physico-Mathematica* of *Mersennus*, wherein I am often named with honour; and partly by others with whom I then conversed in *Park*, without any ostentation. That no man has a great expectation of any thing that shall proceed from either of you two, I am content to let it be your praise.

Another Argument of my *self-conceit*, you take from my contempt of the writers of *Metaphysiques* and *School-Divinity*. If that be a sign of *self-conceit*, I must confess I am guilty; and if your Geometry had then been published, I had contemned that as much. But yet I cannot see the consequence (unless you lend me your better Logique) from *despising insignificant and absurd language* to *self-conceit*.

And again, in your *Vindicia Academicarum*, you put for *boasting*, that in my *Leviathan* Pag. 180. I would have that Book by entire Sovereignty imposed upon the Universities; and in my Review Pag. 395. That I say of my *Leviathan*, I think it may be profitably printed, and more profitably taught in the University. The cause of my writing that Book, was the consideration of what the Ministers before, and in the beginning of the Civill War, by their preaching and writing did contribute thereunto. Which I saw not onely to tend to the Abatement of the then Civill Power, but also to the gaining of as much thereof as they could (as did afterwards more plainly appear) unto themselves. I saw also that those Ministers, and many other Gentlemen who were of their Opinion, brought their Doctrines against the Civill Power from their Studies in the Universities. Seeing therefore that so much as could be contributed to the Peace of our Country, and the settlement of Sovereign Power without any Army, must proceed from Teaching; I had reason to wish, that Civill Doctrine were truly taught in the Universities. And if I had not thought that mine was such, I had never written it. And having written it, if I had not recommended it to such as had the Power to cause it to be taught,

I had written it to no purpose. To me therefore that never did write anything in Philosophy to show my Wit, but (as I thought at least) to benefit some part or other of mankind, it was very necessary to commend my Doctrine to such men as should have the Power and Right to Regulate the Universities. I say my Doctrine; I say not my *Leviathan*. For wiser men may so digest the same Doctrine as to fit it better for a publique teaching. But as it is, I believe it hath framed the minds of a thousand Gentlemen to a conscientious obedience to present Government, which otherwise would have wavered in that Point. This therefore was no vanishing, but a necessary part of the business I took in hand. You ought also to have considered, that this was said in the cloist of that part of my Book which concerneth Policy meerly Civill. Which part if you the Astronomer, that now think the Doctrine unworthy to be taught, were pleased once to honour with praises printed before it, you are not very constant, nor ingenuous. But whether you did so or not, I am not certain, though it was told me for certain. If it were not you, it was some body else whose Judgement has as much weight at least as yours.

And for any thing you have to say from your own knowledge, I remember not that ever I saw either of your faces. Yet you the Professor of Geometry go about obliquely to make me believe that *Vindex* hath discoursed with me, once at least, though I remember it not. I suppose it therefore true; But this I am sure is false, that either he or any man living did ever hear me brag of my Science, or praise my self, but when my defence required it. Perhaps some of our Philosophers that were at Paris at the same time, and acquainted with the same Learned men that I was acquainted with, might take for bragging the maintaining of my opinions, and the not yielding to the reasons alledged against them. If that be ostentation, they tell you the truth. But you that are so wise should have considered, that even such men as profess Philosophy are carried away with the passions of Emulation and Envy (the sole ground of this your accusation) as well as other men, and instanced in your selves. And this is sufficient to shake off your aspersions of *Offension* and *Self-conceit*. For if I added, that my acquaintance know that I am naturally of modest rather then of boasting speech, you will not believe it, because you distinguish not between that which is said upon provocation, and that which is said without provocation, from vainglory.

The next accusation is, That I will be very angry with all men that do not presently submit to my disputes; and that for advancing the reputation of my own skill, I care not what unworthy reflections I cast on others. This is in the Epistle placed before the *Vindicia Academiae*, subscribed by N S, as the Plain Song for H D in the rest of the Book to Descant upon. I know well enough the Authors names; and am sorry that N S has lent his name to be abused to so ill a purpose. But how does this appear? What Argument, what witness is there of it? You offer none; nor am I conscious of any. I begin to suspect, since you the Professor of Geometry have in your objections to the 20 Chapter these words concerning *Vindex*, *Ocularis ille testis de quo hic agitur, erat, ni fallor, ille ipse*; That *Vindex* himself, in other company, has butow'd a visit on me. Seeing you will have me believe it, let it be so; and (as it is likely) not long after my return into England. At which time (for the reputation, it seems, I had gotten by my boasting) divers persons that professed to love Philosophy and Mathematicques, came to see me; and some of them to let me see them, and hear and applaud what they applauded in themselves. I see now it hath happened to me with *Vindex*, as it happened to Dr. Harvey with *Moranus*. *Moranus* a Jesuite came out of Flanders hither, specially (as he says) to see what learned men in Divinity, Ethiques, Physiques, and Geometry were here yet alive, to the end that by discoursing with them in these Sciences, he might correct either his own, or their errors. A most other she was brought (he says) to that most civill and renowned old man Dr. Harvey. That's very well. And in good earnest if he had made good use of the Time which was very patiently afforded him, he might have learned of him (or of no man living) very much knowledge concerning the Circulation of the blood, the Generation of living Creatures, and many

other difficult Points of naturall Philosophy. And if he had had any thing in him but common and childish learning, he could have showed it no where more to his advantage, then before him that was so great a Judge of such matters. But what did he? That pretious time (which was but little, because he was to depart again presently for Flanders) he bestowed wholly in venting his own childish Opinions, not suffering the Doctor scarce to speak; losing thereby the benefit he came for, and discovering that he came not to hear what others could say, but to show to others how learned he was himself already. Why else did he take so little time, and so mispend it? Or why returned he not again? But when he had talked away his time, and found (though patiently and civilly heard) he was not much admired, he took occasion (writing against me) to be revenged of Dr. Harvey, by sleighting his learning publicly; and tells me that his Learning was onely Experiments, which he sayes I say have no more certainty then Civil Histories. Which is false. My words are, *Ante hos nihil certi in Physicâ erat præter Experimenta cuiusjua, & Historias Naturales, sitamen & hæc dicenda certa sent, quæ Civilibus Historiis certiores non sunt.* Where I except expressly from uncertainty the Experiments that every man maketh to himself. But you see the nere-cut, by which vain Glory joyned with Ignorance passeth quickly over to Envy and Contumely.

Thus it seems by your own confession I was used by *Hinc*. He comes with some of my acquaintance in a Visit. What he said I know not, but if he discoursed then, as in his Philosophicall Essay he writeth, I will be bold to say of my self, I was so far from morosity, or (to use his Phrase) from being *tearicall*, as I may very well have a good opinion of my own patience. And if there passed between us the discourse you mention in your *Elenchus*, Pag. 116. it was an incivility in him so great, that without great civility I could not have abstained from bidding him be gone. That which passed between us, you say was this. I complained that whereas I made Sense, nothing but a perception of Motion in the Organ, nevertheless the Philosophy Schools through all Europe led by the Text of Aristotle teach another Doctrine, namely, that Sensation is performed by Species. This is a little mistaken. For I do glory, not complain, that whereas all the Universities of Europe hold Sensation to proceede from Species, I hold it to be a perception of Motion in the Organ. The answer of *Vindex*, you say, was, That the other Hypothesis, whereby Sense was explicated by the Principles of Motion, was commonly admitted here before my Book came out, as having been sufficiently delivered by Des Cartes, Gassendus and Sir Kenelme Digby before I had published anything in this kinde. This then, it seems, was it that made me angry. Truly I remember nor any angry word that ever I uttered in all my life to any man that came to see me, though some of them have troubled me with very impertinent discourse, and with those that argued with me, how impertinently I ever, I always thought it more civility to be somewhat earnest in the defence of my opinion, then by obstinate and affected silence to let them see I contemned them, or hearkned not to what they said. If I were earnest in making good, that the manner of Sensation by such Motion as I had explicated in my *Leviathan*, is in none of the Authors by him named, it was not Anger, but a care of not offending him, with any signe of the contempt which his discourse deserved. But it was Incivility in him to make use of a Visit, which all men take for a profession of Friendship, to tell me that that which I had already published for my own, was found before by Des Cartes, Gassendus, and Sir Kenelme Digby. But let any man read Des Cartes, he shall finde, that he attributeth no Motion at all to the object of sense, but an inclination to action, which inclination no man can imagine what it meaneth. And for Gassendus, and S. Kenelme Digby, it is manifest by their writings, that their opinions are not different from that of *Epictetus*, which is very different from mine. Or if these two, or any of those I conversed with at Paris, had prevented me in publishing my own Doctrine, yet since it was there known, and declared for mine by *Mersennus* in the Preface to his *Ballistica* (of which the three first leaves are employed wholly in the setting forth of my opinion concerning sense, and the rest of the faculties of the Soul) they ought not therefore to be said to have found it out before me. And consequently this answer which

which you say was given me by *Vindex* was nothing else but Untruth and Envy; and (because it was done by way of Visit) Incivility. But you have not alledged, nor can alledge any words of mine, from which can be drawn that I am so angry as you say I am with those that submit not to my *Disputes*. Though the discipline of the University be never so good; yet certainly this behaviour of yours and his are no good Arguments to make it thought so. But you the Professor of Geometry, that out of my words spoken against *Vindex* in my 20 Chapter, argue my angry humour; do just as well, as when (in your *Aritmetica Infinitum*) from the continuall increase of the exels of the row of Squares above the third part of the aggregate of the greatest, you conclude they shall at last be equall to it. For though you knew that *Vindex* had given me first the worst words that possibly can be given, yet you would have that return of mine to be a Demonstration of an angry humour; not then knowing what I told you even now in the beginning of this Lesson, of the sentence given by *Vespasian*. But to this Point I shall speak again hereafter.

Your third Accusation is, That I had my doctrine of Vision, which I pretend to be my own, out of papers which I had a long time in my hands of Mr. Warners. I never had sight of any of Mr. Warners papers in all my life but that of Vision by Refraction (which by his approbation I carryed with me to Paris, and caused it to be printed under his own name, at the end of *Mersennus* his *Cogitata Physico-Mathematica*, which you may have there seen) and another Treatise of the Proportions of Alloy in Gold and Silver coines, which is nothing to the present purpose. In all my conversation with him, I never heard him speak of any thing he had written, or was writing *de Penicillo optico*. And it was from me that he first heard it mentioned that Light and Colour were but Fancy. Which he imbraced presently as a truth, and told me it would remove a rub he was then come to in the discovery of the place of the Image. If after my going hence he made any use of it (though he had it from me, and not I from him) it was well done. But wheresoever you finde my Principles, make use of them, if you can, to demonstrate all the Symptomes of Vision; and I will do (or rather have done and mean to publish) the same; and let it be judged by that, whether those Principles be of mine, or other mens invention. I give you time enough, and this advantage besides, that much of my *Optiques* hath been privately read by others. For I never refused to lend my papers to my friends, as knowing it to be a thing of no prejudice to the advancement of Philosophy, though it be (as I have found it since) some prejudice to the advancement of my own reputation in those Sciences; which reputation I have alwayes postposed to the common benefit of the studious.

You say further (you the Geometrician) that I had the Proposition of the Spirall Line equall to a Parabolical Line from Mr. Robervall. True. And if I had remembered it, I would have taken also his demonstration, though if I had published his, I would have suppressed mine. I was comparing in my thoughts those two Lines, Spirall and Parabolical, by the Motions wherewith they were described; and considering those Motions as uniform, and the Lines from the Centre to the Circumference, not to be little Parallelograms, but little Sectors, I saw that to compound the true Motion of that Point which described the Spirall, I must have one Line equall to half the Perimeter, the other equall to half the Diameter. But of all this I had not one word written. But being with *Mersennus* and Mr. Robervall in the Cloister of the Convent, I drew a Figure on the wall, and Mr. Robervall perceiving the deduction I made, told me that since the Motions which make the Parabolical Lines are one uniform, the other accelerated, the Motions that make the Spirall must be so also; Which I presently acknowledged; and he the next day, from this very method brought to *Mersennus* the demonstration of their equality. And this is the story mentioned by *Mersennus*, Prop. 25. Corol. 2. of his *Hydraulica*; Which I know not who hath most magnanimously interpreted to you in my disgrace.

The fourth accusation is, That I have injured the Universities. Wherein? First, In that I would have the Doctrine of my Leviathan by entire Sovereignty be imposed on them. You often upbraid me with thinking well of my own Doctrine; and grant by consequence, that I thought this Doctrine good. I desired not therefore that any thing should be imposed upon them.

but what (at least in my opinion) was good both for the Common-wealth and them. Nay more I would have the State make use of them to uphold the civill Power, as the Pope did to uphold the Ecclesiasticall. Is it not absurdly done to call this an Injury? But to question (you will say) whether the Civill Doctrine there taught, be such as it ought to be, or not, is a disgrace to the Universities. If that be certain, it is certain also that those Sermons and Books, which have been preached and published, both against the former and the present Government, directly or obliquely, were not made by such Ministers and others as had their breeding in the Universities; though all men know the contrary. But the Doctrine which I would have to be taught there, what is it? It is this, *That all men that live in a Common-wealth, and receive protection of their lives and fortunes from the Supreme Governour thereof, are reciprocally bound as far as they are able, and shall be required, to protect that Governour.* Is it, think you, an unreasonable thing to impose the teaching of such Doctrine upon the Universities? Or will you say they taught it before, when you know that so many men which came from the Universities to preach to the People, and so many others that were not Ministers did stir the People up to resist the then Supreme Civill Power? And was it not truly therefore said, that the Universities receiving their Discipline from the Authority of the Pope, were the Shops, and Operatories of the Clergy? Though the Competition of the Papall and Civill power be taken now away, yet the Competition between the Ecclesiasticall and the Civill power hath manifestly enough appeared very lately. But neither is this an upbraiding of an University (which is a Corporation or Body Artificial) but of particular men that desire to uphold the Authority of a Church, as of a distinct thing from the Common-wealth. How would you have exclaimed, if instead of recommending my *Leviathan* to be taught in the Universities, I had recommended the erecting of a New and Lay-University, wherein Lay-men should have the reading of *Physicks, Mathematicks, Morall Philosophy, and Politicks*, as the Clergy have now the sole teaching of Divinity? Yet the thing would be profitable, and tend much to the Polishing of mans nature, without much publique charge. There will need but one House, and the endowment of a few Professions. And to make some learn the better, it would do very well that none should come thither sent by their Parents, as to a Trade to get their living by, but that it should be a place for such ingenuous men, as being free to dispose of their own time, love Truth for it self. In the mean time Divinity may go on in *Oxford* and *Cambridge* to furnish the Pulpit with men to cry down the Civil Power, if they continue to do as they did. If I had (I say) made such a Motion in my *Leviathan*, though it would have offended the Divines, yet it had been no injury. But 'tis an injury (you will say) to deny in generall the utility of the Antient Schooles, and to deny that we have received from them our Geometry. True, if I had not spoken distinctly of the Schools of Philosophy, and said expressly, that the Geometricians passed not then under the name of Philosophers; and that in the School of *Plato* (the best of the Antient Philosophers) none were received that were not already in some measure Geometricians. *Euclide* taught Geometry; but I never heard of a Sect of Philosophers, called *Eucledians*, or *Alexandrians*, or ranged with any of the other Sects, as *Peripatetiques, Stoiques, Academiques, Epicureans, Pyrrhonians*, &c. But what is this to the Universities of Christendome? Or why are we beholding for Geometry to our Universities, more then to *Gresham Colledge*, or to private men in *London, Paris*, and other places, which never taught or learned it in a publique School? For even those men that living in our Universities have most advanced the *Mathematiques*, attained their knowledge by other means then that of publique Lectures, where few Auditors, and those of unequall proficiency cannot make benefit by one and the same Lesson. And the true use of publique Professors, especially in the *Mathematiques*, being to resolve the Doubts, and Problems (as far as they can) of such as come unto them with desire to be informed.

That the Universities now are not regulated by the Pope, but by the Civill Power, is true, and well. But where say I the contrary? And thus much for the first injury.

Another (you say) is this, that in my *Leviathan* Pag. 380. I say, *The principall Schooles were ordained for the three Professions of Roman Religion, Roman Law, and the Art of Medicine.* Thirdly, that I say, *Philosophy had no otherwise place there then as a hand-maid to Roman Religion.* Fourthly, *since the authority of Aristotle was received there, that Study is not properly Philosophy, but Aristotelicity.* Fifthly, *That for Geometry, till of late times it had no place there at all.* As for the second, it is too evident to be denied; the Fellowships having been all ordained for those Professions; and (having the Change of Religion) being to yet. Nor hath this any Reflection upon the Universities, either as they now are, or as they then were, seeing it was not in their own power to endow themselves, or to receive other Laws and Discipline then their Founder, and the State were pleased to ordain. For the third, it is also evident. For all men know that none but of the Roman Religion had any Stipend or preferment in any University, where that Religion was established; No, nor for a great while, in their Common-wealths; but were every where persecuted as Heretiques. But you will say the words in my *Leviathan* are not, *Philosophy had no place, but hath no place.* Are you not ashamed to lay to my charge a mistake of the word *hath* for *had*? Which was either a mistake of the Printer, or if it were so in the Copy, it could be no other then the mistake of a letter in the writing, unless you think you can make men believe that after fifty years being acquainted with what was publicly profest and practised in *Oxford* and *Cambridge*, I knew not what Religion they were of. Thus taking of advantage from the mistake of a word, or of a letter, I finde also in the *Etenchus*, where for *pretendit scire*, there is *pretendit scire*, which you the Geometrician sufficiently mumble, mistaking it I think for an Anglicisme, not for a fault in the impression.

To the fourth, you pretend, that men are not now so tied to *Aristotle* as not to enjoy a liberty of *Philosophising*, though it were otherwise when I was conversant in *Magdalen Hall*. Was it so then? Then am I absolved, unless you can shew some publique Act of the University made since that time to alter it. For it is not enough to name some few particular ingenious men that usurpe that Liberty in their private discourses, or (with connivence) in their publicke disputations. And your Doctrine, that even here you avow, of *Abstracted Essences*, *Immateriall Substances*, and of *Nunc-stans*; and your improper language in using the word (not as mine, for I have it no where) *Successive Eternity*; as also your Doctrine of *Condensation*, and your arguing from naturall reason the incomprehensible Mysteries of Religion, and your Malicious Writings, are very shrewd signes, that you your selves are none of those which you say do freely *Philosophise*, but that both your Philosophy and your Language are under the Servitude, not of the Roman Religion, but of the Ambition of some other Doctors, that seek, as the Romish Clergy did, to draw all humane learning to the upholding of their Power Ecclesiasticall. Hitherto therefore there is no injury done to the Universities. For the fifth, you grant it, namely, *That till of late there was no allowance for the teaching of Geometry.* But least you should be thought to grant me any thing, you say, you the Astronomer, *Geometry hath now so much place in the Universities, that when Mr. Hebs shall have published his Philosophicall and Geometricall pieces, you assure your self you shall be able to finde a great number in the University, who will understand as much or more of them, then he desireth they should,* &c. But though this be true of the now, yet it maketh nothing against my then. I know well enough that Sir *Henry Saviles* Lectures were founded and endowed since. Did I deny then, that there were in *Oxford* many good Geometricians? But I deny now, that either of you is of the Number. For my Philosophicall and Geometricall pieces, are published, and you have understood onely so much in them, as all men will easily see by your objections to them, and by your own published Geometry, that neither of you understand any thing either in Philosophy or in Geometry. And yet you would have those Books of yours to stand for an Argument, and to be an Index of the Philosophy and Geometry to be found in the Universities. Which is a greater injury and disgrace to them; then any words of mine though interpreted by your selves.

Your

Your last and greatest accusation, or rather railing (for an Accusation should contain, whether true or false, some particular fact, or certain words, out of which it might seem at least to be inferred) is, that I am an enemy to Religion. Your words are, *It is said that Mr Hobbs is no otherwise an Enemy to the Roman Religion, saving onely as it hath the name of Religion.* This is said by *Vindex*. You the Geometrician in your Epistle Dedicatory say thus, *With what pride and imperiousness he tramples on all things both Humane and Divine, uttering fearfull and horrible words of God, (peace) of Sin, of the Holy Scripture, of all Incorporeal substances in generall, of the Immortall soul of man, and of the rest of the weighty points of Religion (down) it is not so much to be doubted as lamned.* And at the end of your objections to the 18 Chapter, *Perhaps you take the whole History of the Fall of Adam for a Fable, which is no wonder, when you say the Rules of honouring and worshipping of God are to be taken from the Law.* Down I lay; you baite now at the Supreme Legislative Power. Therefore it is not I, but the Laws which must rate you off. But do not many other men as well as you read my *Leviathan*, and my other Books? And yet they all finde not such enmity in them against Religion. Take heed of calling them all Atheists that have read and approved my *Leviathan*. Do you think I can be an Atheist and not know it? Or knowing it durst have offered my Atheism to the Press? Or do you think him an Atheist, or a contemner of the Holy Scripture, that sayeth nothing of the Deity, but what he proveth by the Scripture? You that take so hainously that I would have the Rules of Gods worship on a Christian Common-wealth taken from the Laws, tell me, from whom you would have them taken? From your selves? Why so, more then from me? From the Bishops? Right, if the Supreme Power of the Common-wealth will have it so; If not, why from them rather then from me? From a Consistory of Presbyters by themselves, or joyned with Lay-Elders, whom they may sway as they please? Good, If the Supreme Governour of the Common-wealth will have it so. If not, why from them, rather then from me, or from any man else? They are wiser and learnedier then I. It may be so; but it has not yet appeared. Howsoever, let that be granted. Is there any man so very a fool as to subject himself to the Rules of other men in those things which so nearly concern himself, for the Title they assume of being wise and learned, unless they also have the sword which must protect them. But it seems you understand the sword as comprehended. If so, do you not then receive the Rules of Gods worship from the Civill Power? Yes doubtless; and you would expect, if your Consistory had that sword, that no man should dare to exercise or teach any Rules concerning Gods worship which were not by you allowed. See therefore how much you have been transported by your malice towards me, to injure the Civill Power by which you live. If you were not despised, you would in some places and times, where and when the Laws are more severely executed, be shipped away for this your madness to *America*, I would say, to *Anticyra*. What luck have I, when this, of the Laws being the Rules of Gods publique worship, was by me said and applied to the Vindication of the Church of *England* from the Power of the Roman Clergy, it should be followed with such a storm from the Ministers Presbyterian and Episcopall of the Church of *England*? Again, for those other Points, namely, that I approve not of Incorporeall Bodies; nor of other Immortality of the soul, then that which the Scripture calleth Eternal Life, I do but as the Scripture leads me. To the Texts whereof by me alledged, you should either have answered, or else forborne to revile me for the conclusions I derived from them. Lastly, what an absurd question is it to ask me whether it be in the Power of the Magistrate, whether the world be eternall or not? It were fit you knew it is in the Power of the Supreme Magistrate to make a Law for the punishment of them that shall pronounce publicly of that question any thing contrary to that which the Law hath once pronounced. The truth is, you are content that the Papall power be cut off, and declaimed against as much as any man will; but the Ecclesiasticall Power which of late was aimed at by the Clergy here, being a part thereof, every violence done to the Papall Power is sensible to them yet; like that which I have heard say of a man, whose leg being cut off for prevention of a Gangrene that began in his Toe, would nevertheless complain of a pain in his Toe, when his leg was cut off.

Thus

Thus much in my defence; which I believe if you had foreseen, this Accusation of yours had been left out. I come now to examine (though it be done in part already) what Manners those are which I finde every where in your Writings.

And first, How came it into your minds that a man can be an Atheist, I mean an Atheist in his Conscience? I know that *David* confesseth of himself, upon sight of the Prosperity of the wicked, that his feet had almost slip'd; that is, that he had slip't into a short doubtfulness of the Divine Providence. And if any thing else can cause a man to slip in the same kinde, it is, the seeing such as you (who though you write nothing, but what is dictated to each of you by a Doctor of Divinity) to break the greatest of Gods Commandment (which is Charity) in every Line before his face. And though such forgettings of God be somewhat more then short doubtings, and sudden transportations incident to humane Passion, yet I do not for that cause think you Atheists, and enemies of Religion, but only ignorant and imprudent Christians. But how, I say, could you think me an Atheist, unless it were because finding your doubts of the Duty more frequent then other men do, you are thereby the apter to fall upon that kinde of reproach? Wherein you are like women of poor & evil Education when they scold; amongst whom the readiest disgracefull word is *Whore*. Why not Thief, or any other ill name, but because when they remember themselves, they think that reproach the likeliest to be true?

Secondly, tell me what crime it was which the Latines called by the name of *Scelus*? You think not (unless you be Stoiques) that all Crimes are equall. *Scelus* was never used but for a Crime of greatest mischief, as the taking away of Life and Honour; and besides, basely acted, as by some clandestine way, or by such a way as might be covered with a Lye. But when you insinuate in a writing publiht that I am an Atheist, you make your selves Authors to the multitude, and do all you can to stir them up to attempt upon my life; and if it succeed, then to sneak out of it by leaving the fault on them that are but actors. This is to endeavour great mischief basely; and therefore *Scelus*. Again, to deprive a man of the honour he hath merited, is no little wickedness; and this you endeavour to do by publishing falsly that I challenge as my own the inventions of other men. This is therefore *Scelus*. Publickly to tell all the world that I will be angry with all men that do not presently submit to my Dictates; to deprive me of the friendship of all the world. Great damage, and a lie, and yours. For to publish any unwruth of another man to his disgrace, on hear-say from his enemy, is the same fault as if he publiht it on his own credit. As I should say I have heard that Dr. *Wallis* was esteem'd at *Oxford* for a simple fellow, and much inferior to his fellow-professor Dr. *Ward* (as indeed I have heard, but do not believe it) though this be no great disgrace to Dr. *Wallis*, yet he would think I did him injury. Therefore publique Accusation upon hear-say is *Scelus*. And whosoever does any of these things does *Scelerate*. But you the Professors of the Mathematicques at *Oxford*, by the advice of two Doctors of Divinity have dealt thus with me. Therefore you have done (I say not foolishly, though no wickedness be without folly, but *Scelerate*, *ὄργη ἐστὶν δειλότης*).

Thirdly, it is ill Manners, in reprehending truth, to send a man in a boasting way to your own errors; as you the Professor of Geometry have often sent me to your two Tractates of the *Angle of Contact*, and *Arithmetica Infinitorum*.

Fourthly, it is ill Manners, to diminish the just reputation of worthy men after they be dead, as you the Professor of Geometry have done in the case of *Joseph Scaliger*.

Fifthly, when I had in my *Leviathan* suffered the Clergy of the Church of England to escape, you did imprudently in bringing any of them in again. An *Ulysses* upon so light an occasion would not have ventured to return again into the Cave of *Polypheemus*.

Lastly, how ill does such Levity and Scurrility, which both of you have shewn so often in your writings, become the gravity and sanctity requisite to the calling of the Ministry? They are too many to be repeated. Do but consider you the Geometrician how unhandsome it is to play upon my name, when both yours and mine are plebeian names; though from *Willis* by *Wallis*, you go from yours in *Wallisius*. The jest of using at every word *Me Hobbi*, is lost to them beyond Sea. But this is not so ill as some of the rest. I will write out one of them as it is in

the fourth Page of your *Elenchus*; Whence it appears that your *Empusa* was of the number of those *Fairies* which you call in English Hob-goblins. The word is made of *ev* and *pus*; and whence comes the childrens play called the play of *Empusa*, *Anglice* (hitherto in Latine all but Hob-goblins; then follows) *Fox, Fox, come out of your hole* (in English, then in Latine again,) in which the boy that is called the *Fox*, holds up one foot, and jumps with the other, which in English is to hop. When a stranger shall read this, and hoping to finde therein some witty conceit, shall with much adoe have gotten it interpreted and explained to him, what will he think of our Doctors of Divinity at Oxford, that will take so much pains as to go out of the language they set forth in, for so ridiculous a purpose? You will say it is a pretty *Paronomasia*. How you call it there I know not, but it is commonly called here a *Clinch*; and such a one as is too insipide for a boy of twelve years old, and very unfit for the sanctity of a Minister, and gravity of a Doctor of Divinity. But I pray you tell me where it was you read the word *Empusa* for the boys play you speak of, or for any other play amongst the Greeks. In this (as you have done throughout all your other writings) you presume too much upon your fast cogitations. There be a hundred other scoffing passages, and ill-favoured attributes given me in both your writings, which the Reader will observe without my pointing to them, as easily as you would have him; and which perhaps some young Students, finding them full of Gall, will mistake for Salt. Therefore to disabuse those youngmen, and to the end they may not admire such kind of wit, I have here and there been a little sharper with you then else I would have been. If you think I did not spare you, but that I had not wit enough to give you as scornfull names as you give me, are you content I should try? Yes (you the Geometrician will say) give me what names you please, so you call me not *Arithmetica Infinitorum* (I will not.) Nor *Angle of Centa*, nor *Arch-Spirall*, Nor *Quotient* (I will not.) But I here dismiss you both together. So go your wayes, you uncivill Ecclesiastiques, Inhumane Divines, Dedottors of Morality, Unafinow Colleagues, Egregious pair of Ishmaels, most wretched Vindices and Indices *Academiarum*; and remember *Vespasian's* Law, that it is uncivill to give ill language first, but civill and lawfull to return it. But much more remember the Law of God, to obey your Sovereigns in all things; and not only not to derogate from them, but also to pray for them, and as far as you can to maintain their Authority, and therein your own protection. And (do you hear?) take heed of speaking your minde so clearly in answering my Leviathan, as I have done in writing it. You should do best not to meddle with it at all, because it is undertaken, and in part published already, and will be better performed, from Term to Term, by one Christopher Pike.

